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Technical note

Optimal phase difference reconstruction: comparison of two methods

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Abstract

The present study compares the performance of the weighted mean (WM) and sensitivity encoding (SENSE) methods for reconstructing phase difference images over a large range of signal-to-noise ratio (SNR). It is found that the WM algorithm is suboptimal, compared to the SENSE method at low SNR. Numerical simulations, phantom and in vivo results are presented.

Keywords: Phase contrast; SENSE; Phased-array; B_0 field mapping

1. Introduction

In B_0 field mapping and phase contrast imaging, two images are acquired at different echo times (TE), and the phase difference between the two is calculated. The phase difference contains information relating to the flow rate, temperature or the field perturbation that can be useful clinically.

Although a single large-volume coil can be used in these applications, multiple small-surface coils, such as a phased-array coil, can offer a signal-to-noise ratio (SNR) advantage without sacrificing the spatial coverage. Combining multiple phase differences measured by a phased-array coil into a single phase difference is complicated by the fact that each coil has its own intrinsic phase variation. The simple average will not correctly account for this variation. Previously proposed methods include the weighted mean (WM), whereby the phase of a weighted sum of the product of complex images is calculated, and sensitivity encoding (SENSE) reconstruction, whereby an optimal coil-combined image is calculated prior to calculating the phase difference [1,2]. A previous study concluded there is no significant difference between the two methods at sufficiently high SNR, although a systematic analysis of the performance with respect to SNR was not performed [3].

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The goal of the present study is to investigate the performance of the WM and SENSE as a function of the SNR of the images. Numerical simulations, phantom and in vivo results are presented.

2. Theory

Let each pixel of the complex image from the first echo acquired by the ith coil be given by Eq. (1):

$$F_i = C_i f + n_i \tag{1}$$

where C_i is the coil sensitivity, f is the effective proton density and n_i is Gaussian noise with standard deviation σ in the real and imaginary parts. Likewise, let each pixel of the image from the second echo be given by Eq. (2):

$$S_i = C_i s + m_i \tag{2}$$

where s is the effective proton density and m_i is noise. The WM method for determining the phase difference between f and s is given by Eq. (3):

$$\Delta\Phi_{\mathrm{WM}} = angle\left(\sum F_i \bar{S}_i\right) \tag{3}$$

where the overbar denotes complex conjugation and the summation is over coils [1]. Note that Eq. (3) is comparable to Eq. (13) of Ref. [1] for the definition of WM but differs from Eq. (3) of Ref. [3], which uses $\sum F_i \bar{S}_i / \sum |F_i||S_i|$;

however the angle of this expression is identical to Eq. (3) so the parameter of importance (i.e., the angle) is consistent between the definitions.

The SENSE method is based on techniques for optimal coil combination [2,4–6], also known as parallel imaging or matched filtering. The expression is given by Eq. (4):

$$\Delta\Phi_{\mathrm{SENSE}}=angle(F\overline{S}) \quad (4)$$

where F and S are the optimal coil-combined images given by $F=f+\sum\overline{C}_i n_i/\sum\overline{C}_j C_j$ and $S=s+\sum\overline{C}_i m_i/\sum\overline{C}_j C_j$. In practice, F and S can only be obtained modulated by some combination of the coils, e.g., C_iF and C_iS [6], so the phase difference must be calculated from $\sum\overline{C}_iC_iF\overline{S}$. Note the coil sensitivity contamination is only present in the amplitude part so the phase difference is unaffected.

Under the assumption that $|f|$ and $|s|$ are greater than 2σ, a previous study found that the two methods are identical [3]. However, without making any assumptions, the expressions $\sum F_i\overline{S}_i$ and $\sum\overline{C}_iC_iF\overline{S}$ can be found to be identical only up to first order in the noise terms, by substituting the expressions from Eqs. (1) and (2). They differ in the second-order noise terms, which are $\sum n_i\overline{m}_i$ and $\sum\overline{C}_in_i\sum C_j\overline{m}_j/\sum\overline{C}_kC_k$, respectively. In the general case, these expressions are analytically intractable, although in simple examples and numerical simulations, it can be verified that the former has a higher variance. Since the variance of the angle is directly related to the SNR of the base expression [7], it follows that the variance of $\Delta\Phi_{\mathrm{WM}}$ is higher than that of $\Delta\Phi_{\mathrm{SENSE}}$.

A simple example illustrates the result. Letting C_1=C_2=1, the expressions $\sum n_i\overline{m}_i$ and $\sum\overline{C}_in_i\sum C_j\overline{m}_j/\sum\overline{C}_kC_k$ evaluate to $(n_1\overline{m}_1+n_2\overline{m}_2)$ and $\frac{1}{2}(n_1\overline{m}_1+n_1\overline{m}_2+n_2\overline{m}_1+n_2\overline{m}_2)$, respectively. If the noise is uncorrelated, then the individual products $n_i\overline{m}_j$ have zero mean and variance σ^4 [8]; note that $\frac{1}{2}n_i\overline{m}_j$ has zero mean and variance $1/4\sigma^4$. Making use of the fact that the variance of a sum is equal to the sum of the variances, the variance of $(n_1\overline{m}_1+n_2\overline{m}_2)$ is evaluated to be $2\sigma^4$, whereas the variance of $\frac{1}{2}(n_1\overline{m}_1+n_1\overline{m}_2+n_2\overline{m}_1+n_2\overline{m}_2)$ is just σ^4. Thus, the variance of $\Delta\Phi_{\mathrm{WM}}$ is higher than the variance of $\Delta\Phi_{\mathrm{SENSE}}$.

3. Methods

Data were acquired on a 3.0 T EXCITE scanner using an eight-channel head coil (GE Healthcare, Milwaukee, WI, USA). Experiments were performed on a uniform agar gel phantom and a human volunteer. The B_0 field mapping protocol used measurements at two TEs with a gradient-echo pulse sequence (TE 3.2 and 5.5 ms, matrix size 128×128, field of view 22 cm, TR 500 ms, slice thickness 1 mm). Complex images from all coils were saved, and the combined phase difference was estimated using the WM and SENSE methods. In order to vary SNR, the flip angle was varied (1°, 2°, 5°, 10°, 20° and 45°), which provided an SNR range from 8 to 185 (phantom) and 7 to 100 (in vivo). Noise was assumed independent and Gaussian-distributed for all coils and echos, although both WM and SENSE can be modified to account for noise correlation [1,2]. It has been reported that properly accounting for noise correlations does not significantly affect the SNR [3,4]. Numerical simulations and data processing were performed in MATLAB (The Mathworks, Natick, MA, USA). Phase unwrapping of $\Delta\Phi_{\mathrm{WM}}$ and $\Delta\Phi_{\mathrm{SENSE}}$ was performed using FSL Prelude software [9]. The performance of the methods was assessed by measuring the variance of the unwrapped phase differences over a defined region of interest, which was obtained by thresholding the amplitude images.

3.1. Implementation

The implementation of SENSE for unit speedup factor used in the present study was summation using profiles estimated from ratios (SUPER) [6]. Image domain convolution with a 9×9 Hamming window was used to smooth the coil sensitivities [10]. An alternative choice to SUPER is the matched filter, which uses the principal eigenvector of the correlation matrix within a local neighborhood to estimate coil sensitivities [5]. The performance of the two methods depends on the window (or neighborhood) size. To attain 90% of the optimal performance, SUPER requires a 9×9 window (81 pixels), whereas the matched filter requires 200 pixels, which translates into faster computation times for SUPER. In other respects, the methods are comparable.

4. Results

Fig. 1 shows a plot of the variance of $\Delta\Phi_{\mathrm{SENSE}}$ (solid line) and $\Delta\Phi_{\mathrm{WM}}$ (dashed line) for the special case C_1=1 and C_2=2. The signals f and s were set equal to 1, and complex Gaussian noise was added to vary the SNR. The SNR was calculated as the square root of the sum of

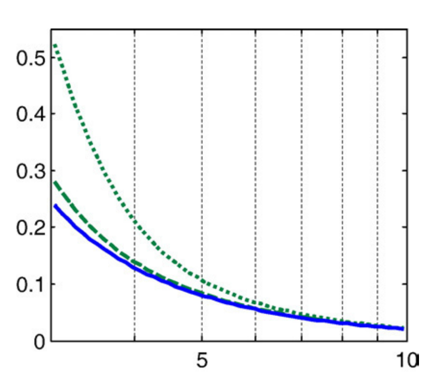

Fig. 1. Simulation results: plot of the variance of $\Delta\Phi_{\mathrm{SENSE}}$ (solid line: two coils and eight coils — lines are coincident) and $\Delta\Phi_{\mathrm{WM}}$ (dashed line: two coils, dotted line: eight coils). The SNR was varied by adding noise.

squares of the coils divided by the standard deviation of the added noise. An important special case is when the number of coils is large but most of the coil sensitivities are negligible, which corresponds with the typical signal obtained from a phased array. To simulate this situation, six coils containing noise only were included (C_{3-8}=0). The variance of $\Delta\Phi_{WM}$ (dotted line) increases with number of coils, whereas the variance of $\Delta\Phi_{SENSE}$ (solid line) remains the same — note the two-coil and eight-coil lines are coincident.

Fig. 2 shows the phantom results. The SNR at the different flip angles were 8, 12, 29, 54, 75, 105 and 185. Panels A and B show $\Delta\Phi_{SENSE}$ and $\Delta\Phi_{WM}$ at the highest SNR. Panels C and D show $\Delta\Phi_{SENSE}$ and $\Delta\Phi_{WM}$ at the lowest SNR. The variance is plotted as a function of SNR in E.

Fig. 3 shows in vivo results. The SNR at the different flip angles were 7, 8, 18, 32, 47, 63 and 100. Panels A and B show $\Delta\Phi_{SENSE}$ and $\Delta\Phi_{WM}$ at the highest SNR. Panels C and D show $\Delta\Phi_{SENSE}$ and $\Delta\Phi_{WM}$ at the lowest SNR. The variance is plotted as a function of SNR in Panel E.

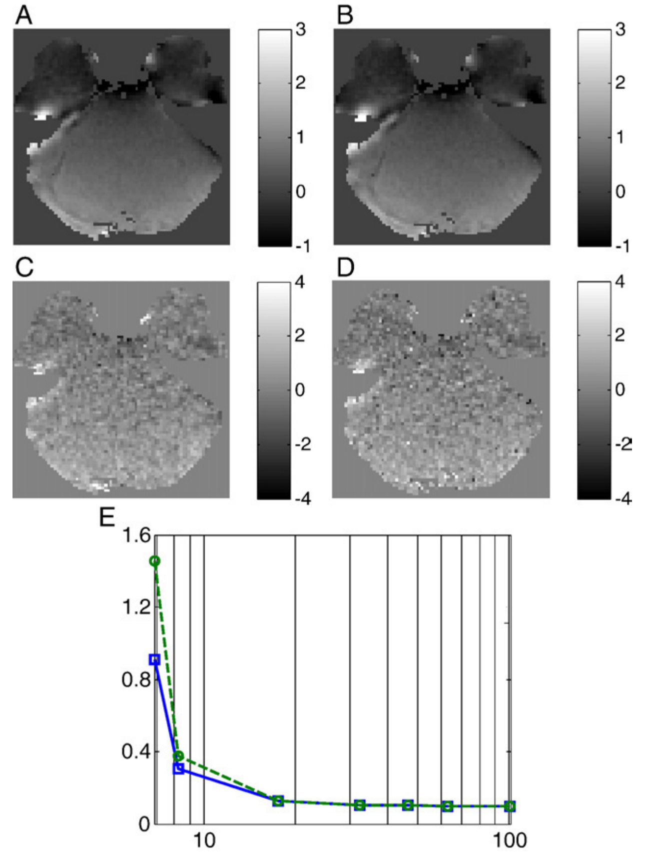

Fig. 3. In vivo results. (A and B) $\Delta\Phi_{SENSE}$ and $\Delta\Phi_{WM}$ at SNR=100. (C and D) $\Delta\Phi_{SENSE}$ and $\Delta\Phi_{WM}$ at SNR=7. Colorbar units are in radians. (E). Variance of $\Delta\Phi_{SENSE}$ (solid line) and $\Delta\Phi_{WM}$ (dashed line) as a function of the SNR. The SNR was varied by changing flip angle.

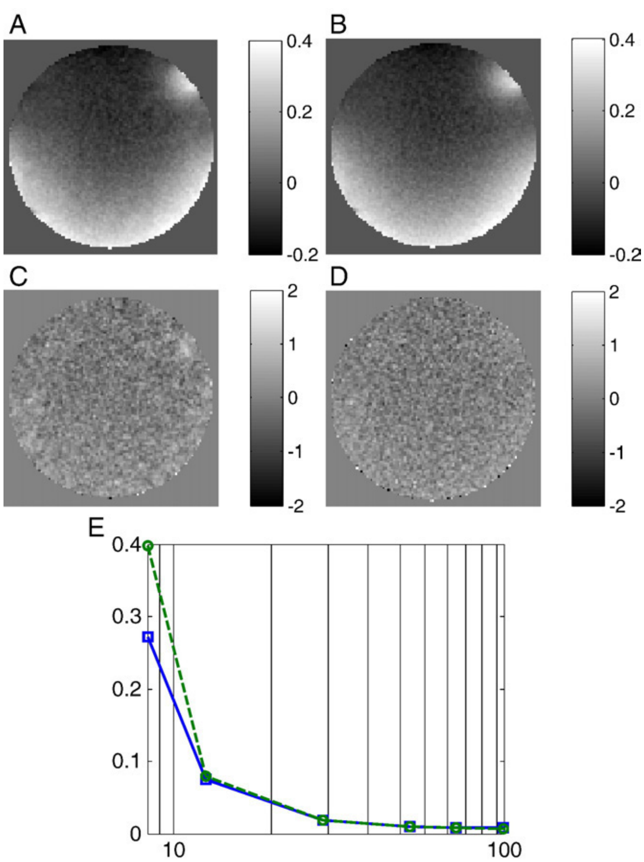

Fig. 2. Phantom results. (A and B), $\Delta\Phi_{SENSE}$ and $\Delta\Phi_{WM}$ at SNR=185. (C and D) $\Delta\Phi_{SENSE}$ and $\Delta\Phi_{WM}$ at SNR=8. Colorbar units are in radians. (E) Variance of $\Delta\Phi_{SENSE}$ (solid line) and $\Delta\Phi_{WM}$ (dashed line) as a function of the SNR. The SNR was varied by changing flip angle.

The simulated and experimental results confirm the theoretical expectation that SENSE and WM perform equally well at high SNR but that the variance of $\Delta\Phi_{WM}$ is higher than the variance of $\Delta\Phi_{SENSE}$ at low SNR. The increase is due larger to second-order noise terms present in $\Delta\Phi_{WM}$. The WM expression [Eq. (3)] performs a weighted summation of the complex images using the images themselves as weights. Theoretically, this should provide the optimal SNR; however, the images contain noise, and hence, the weights are not perfectly known. Thus, the optimal combination is not obtained. Smoothing the weights by low-pass filtering decreases the noise and provides a better estimate of the phase difference, as long as the smoothing process does not corrupt the coil sensitivities [6].

5. Conclusions

Phase differences determined by WM ($\Delta\Phi_{WM}$) and SENSE ($\Delta\Phi_{SENSE}$) phased-array coil combination methods have been compared in the context of B_0 field mapping. The two methods are indistinguishable when the SNR is high, although theoretical and experimental results show $\Delta\Phi_{WM}$ exhibits higher noise variance than $\Delta\Phi_{SENSE}$ at low SNR.

The difference depends on the SNR and number of coils and is most evident at SNR<10 with many coils. The main disadvantage of $\Delta\Phi_{\mathrm{SENSE}}$ is the additional computation time to perform smoothing; however, this is minor and may be unimportant for many applications.

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