BENG280A, Principles of Biomedical Imaging 10/13/04
Fall Quarter 2004

HOMEWORK #3 (Corrected 10/18/04)
Due on Wednesday 10/20/04

Readings:
1. Review chapter 2 as necessary. Read Chapter 6 in Suetens. You can skip sections 6.5.9 and 6.5.10 for now.

Problems:

1. Consider the function $g(x) = \cos^2(2\pi k_0 x)$. You sample this signal in the spatial domain with with a sampling rate $K_S = 1/\Delta x$ (e.g. samples spaced at intervals of Δx). What is the minimum sampling rate that you can use without aliasing?

2. A 2D object has an FOV of 100 cm in the *x* direction and an FOV of 2 cm in the *y* direction. What is the spacing of samples in the Fourier domain that avoids aliasing (i.e. the Nyquist condition)?

3. Consider the 2D object $m(x,y) = \delta(x)\left(\delta(y-L) - \delta(y+L)\right)$ consisting of two impulses. The Fourier transform of the object is sampled in the k_y direction with sampling interval Δk_y. The reconstructed image is obtained from the inverse transform of the sampled spectrum.
   a) At what values of Δk_y will the reconstructed image be equal to zero?
   b) What is the reconstructed image when we sample in the Fourier domain with the function $\sum_{n=-\infty}^{\infty} \delta\left(k_y - \frac{2n+1}{4L}\right)$?

4. A 2D object has an FOV of 19.2 cm in both the *x* and *y* directions. We sample the 2D Fourier transform of the object. If we want to achieve a resolution of *1* mm in the *x* direction and *2* mm in the *y* direction, how should we sample *k*-space? (i.e. give the sampling intervals and the extent of the sampling region).

5. Show that the Fourier transform of $g(ax+b)$ is given by $\frac{1}{|a|} G\left(\frac{k_x}{a}\right) e^{j2\pi k_x b/a}$. Hint: This problem is most easily done by just using the definition of the Fourier transform and using substitution of variables. It's also good to treat the cases of $a > 0$ and $a < 0$ separately.

6. Define the 2D object $m(x,y) = rect(3x/2)rect(2y)$ with 2D Fourier transform $M(k_x,k_y)$. Define $M_1(k_x,k_y) = M(k_x,k_y)comb(k_x/2,k_y)$ and $M_2(k_x,k_y) = M(k_x,k_y)comb((k_x-1)/2,k_y)$. Hint: You may find it useful to use the results of problem 5.
   (a) Derive and sketch the reconstructed objects $m_1(x,y)$ and $m_2(x,y)$
   (b) Derive and sketch the object $m_S(x,y) = m_1(x,y) + m_2(x,y)$
   (c) Derive and sketch the object $m_S(x,y) = m_1(x,y) + 0.5m_2(x,y)$
   (d) Derive and sketch the object $m_S(x,y) = e^{j4\pi y}m_1(x,y) + e^{-j4\pi y}m_2(x,y)$.

MATLAB Exercise on next page.

MATLAB Exercise:.

Steps:

1. First download the file BE280Ahw1im.mat from the course website.
2. Load the image into MATLAB with the command: *load BENG280Ahw1im*.
3. Compute the 2D Fourier transform of the image with the command *Mf = fft2(Mimage);* where the 2D transform will now be stored in the variable *Mf*. Remember to add the semi-colon at the end of the command, otherwise MATLAB will display all the numbers in the matrix! The command *fft2* puts the zero-frequency value of the transform at the first indices of the matrix. For display it's convenient to put the zero-frequency value in the center of the matrix. To do this, type *Mf = fftshift(Mf);*

4. ***Aliasing***

(a) Aliasing in the x-direction. Pick out every other column in the transform matrix and take the inverse transform. The steps are as follows: (the >> represents the MATLAB prompt)

```
>> alias_span = 1:2:256;
>> Mf2 = zeros(256,256);
>> Mf2(:,alias_span) = Mf(:,alias_span);
>> Mf2 = fftshift(Mf2);
>> M_aliasx = ifft2(Mf2);
>> imagesc(abs(M_aliasx)); % This will be an image showing aliasing in the x-direction.
```

(b) **Demonstrate aliasing in the y-direction**. **Hand in code and image.**

(c) **Demonstrate aliasing in the x and y directions**. **Hand in code and image.**

(d) Show one additional example of aliasing, where you take every Nth sample (e.g. every 4^{th} or 8^{th} sample)**.** Show that the resultant image is what you would expect from sampling theory.