BENG280A, Principles of Biomedical Imaging 10/20/03
Fall Quarter 2004

HOMEWORK #4
Due Wednesday November 3, 2004

NOTE: Because of the midterm, you have two weeks to do this homework assignment. Material for problems 7 and 8 will be covered next week.

Readings:

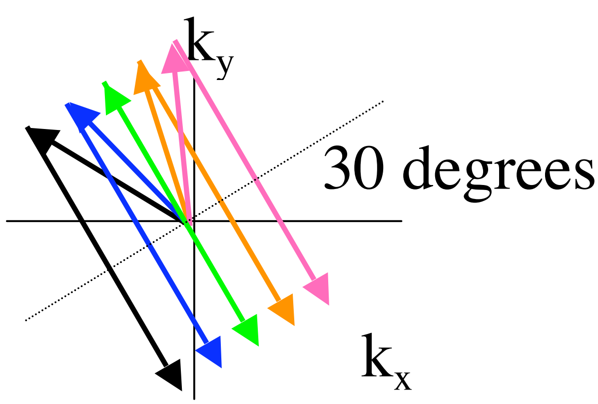
1. Finish Chapter 6 in Suetens.

Problems:

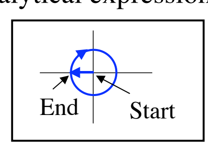
1. Show that $M_z(t) = M_0 + \left(M_z(0) - M_0\right)e^{-t/T_1}$ is a solution to the differential equation $dM_z/dt = \dfrac{M_0 - M_z}{T_1}$.

2. Show that $M(t) = M(0)e^{-j\omega_0 t}e^{-t/T_2}\exp\left(-j\int_0^t \Delta\omega(\vec{r},\tau)d\tau\right)$ is a solution to the differential equation $dM/dt = -(j(\omega_0 + \Delta\omega(\vec{r},t)) + 1/T_2)M$ where $M(t) \equiv M_x(t) + jM_y(t)$. Hint: Use the Fundamental Theorem of Calculus: $\dfrac{d}{dx}\int_0^x f(u)du = f(x)$.

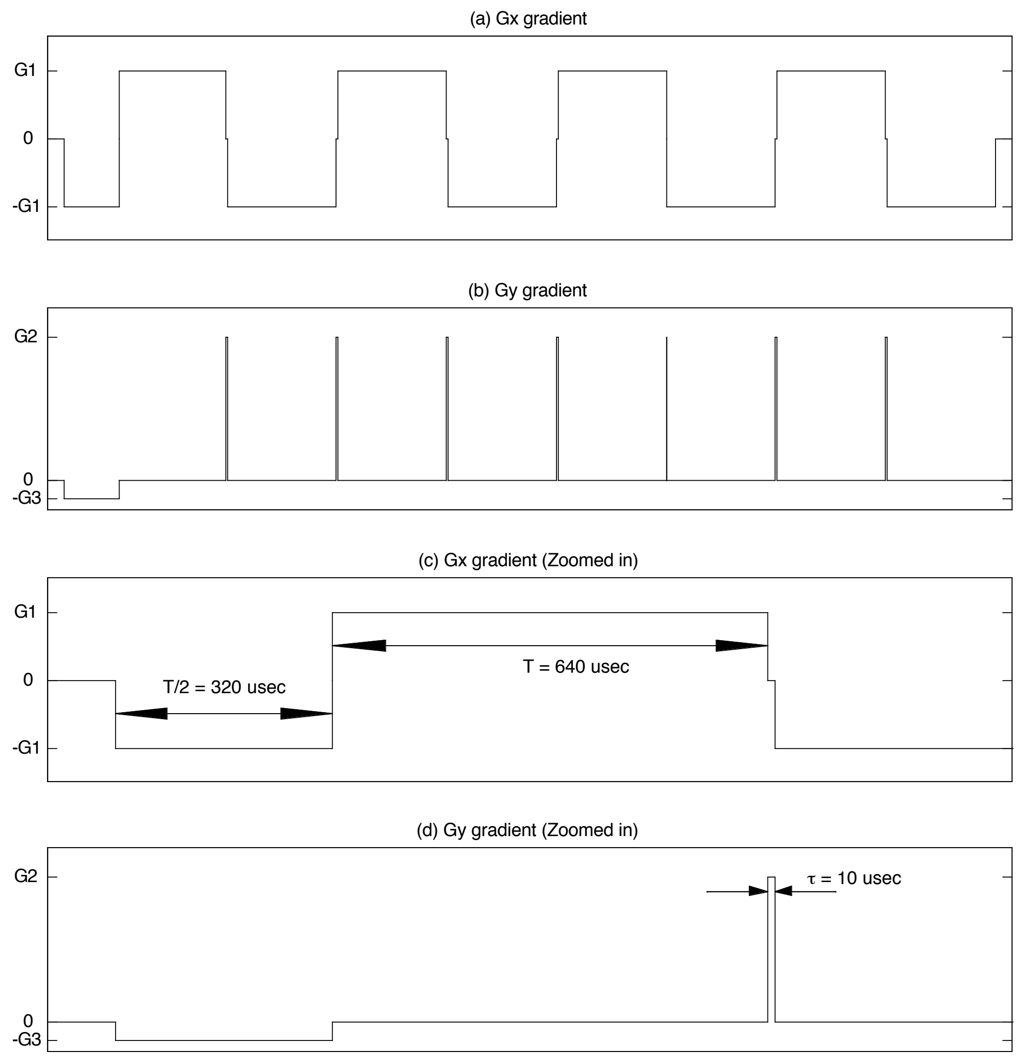
3. Draw a pulse sequence that will achieve the k-space trajectory shown below. Describe a situation where you might use this pulse sequence. **Hint**: this is just the spin-warp sequence in class, rotated by 60 degrees in k-space.



4. Draw a pulse sequence that achieves the k-space trajectory shown and give analytical expressions for the gradient waveforms. Note this starts at the origin and circles clockwise once with radius K. Assume that the time to make the circle is T.



5. Consider the gradient waveforms shown in the figure below. The full waveforms are shown in panels (a) and (b), and zoomed-in views are shown in (c) and (d). The analog-to-digital converter (ADC) is turned on during the flat parts of the readout (Gx) gradients with a sampling rate of Δt.
(a) Draw the k-space trajectory.
(b) Determine the sequence parameters (G1, G2 and G3, and Δt) to achieve the following image specifications: $FOV_x = FOV_y$ = 19.2 cm, δ_x = 3 mm and $\delta_y = 24$ mm.



6. Consider the spin-echo saturation recovery pulse sequence discussed in class. Compute the signal differences between gray matter and white matter for the following conditions You may assume that the T_1 values of gray and white matter are 1300 and 900 ms, respectively, the T_2 values of gray and white matter are 110 and 80 ms, respectively, and that the proton density of gray matter is 1.2 times that of white matter.
   a) Use MATLAB to plot the signal differences for TE 10 ms and TR varying from 1000 to 5000 ms. At what TR is the signal difference maximized?
   b) Use MATLAB to plot the signal differences for TR = 5000 and TE varying from 10 to 500 ms. At what TE is the signal difference maximized?

7. In class, we derived the zeroth and first order moments for a bipolar gradient pulse.
   (a) Derive the 2^{nd} order moment for this pulse.
   (b) Design a bipolar pulse that results in 90 degrees of phase shift for an object with a velocity of 20 cm/sec and zero acceleration. Use a gradient amplitude of 4 Gauss/cm.
   (c) What is the total phase shift using the pulse designed in (b) if the object is also accelerating at 300 cm/sec^2?
   (d) Modify the design of your pulse so that the total phase shift due to both velocity and acceleration is 90 degrees. If necessary, you may adjust the gradient amplitude.

8. Consider a 1D object $m(x) = rect(x)$ with velocity profile $v(x) = xv_o\text{rect}(x)$. The signal integrated over the object is of the form $s = \int_{-\infty}^{\infty} m(x)\exp(j\varphi(v(x))dx$. Find an expression for this signal if the signal is acquired after a bipolar pulse with first order moment M_1. For what values of the first order moment does this signal go to zero? Give an interpretation of your result.