BENG280A, Principles of Biomedical Imaging 11/10/04
Fall Quarter 2004

HOMEWORK #6
Due Wednesday 11/17/04

Readings:
1. Review Suetens chapters 4 and 5 as necessary. Start reading Seutens chapter 7.

Problem 1
In this problem, we will examine the filtered back projection of the two dimensional object $m(x,y) = \cos(2\pi k_0 x)$.

(a) Sketch the object and its projections at theta = 0 , 45 and 90 degrees. Label your sketch.
(b) Prove the identity $\delta(x - x_0, y - y_0) = \frac{\delta(r - r_0)\delta(\theta - \theta_0)}{r}$ where $x_0 = r_0 \cos\theta_0$ and $y_0 = r_0 \sin\theta_0$.
Hint: Remember that two delta functions are equivalent if when integrated with a test function they give the same result. See slide 16 from Linear Systems lecture 1 and the corresponding in-class example (this was done on the blackboard).
(c) Using the identity proved in part (b), derive the 2D Fourier transform of your object in Cartesian coordinates and then convert your answer to polar coordinates.
(d) Using the projection theorem, use the answer from part c to derive an expression for the projection $p_\theta(r)$. Does this agree with your sketch from part (a)?
(e) Find the filtered projection for each angle θ. (e.g. see slide 17 in CT lecture 2)
(f) Use the backprojection formula (Eqn 5.16 in the textbook or Slides 10-12 in CT lecture 2) to backproject the filtered projection and derive the reconstructed object.

Problem 2
Now consider the object $m(x,y) = \cos(2\pi 4x) + \cos(2\pi 4y)$. Using this object, follow steps (a) and (c)-(f) from problem 1.

Problem 3
Consider an X-ray source with a photon density of $\phi = 2.5 \times 10^{10}$ *photons/cm*^{2} */ roentgen*. If the contrast C is 0.1 and the average transmission p is 0.05, what dose is required to achieve a SNR of 50 assuming a 1 mm x 1 mm detector?

Problem 4 MATLAB Exercise
Define a 257x257 object where the center 61x61 square is 1 and the object is zero everywhere else.

(a) Use the radon function to compute the projections for angles from 0 to 180. Try angular increments of 1 degree, 0.5 degree, and 0.25 degrees (e.g. theta = 0:0.5:179.5). Examine the sinograms. Does the projection at 45 degrees agree with what you found in last week's homework?
(b) Use the iradon function to compute the filtered backprojection reconstruction of your image. What is the effect of the varying angular increments?
(c) Experiment with the different filtering and interpolation options in iradon. What is the effect of using a Hamming or Hann filter?