Bioengineering 280A
Principles of Biomedical Imaging

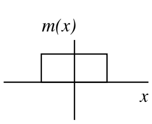
Fall Quarter 2004
MRI Lecture 3

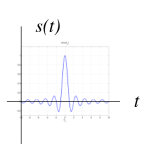
Topics

- Review signal equation
- Sampling requirements
- Slice Selection
- Gradient Echo and Spin Echo
- Image Contrast

MR signal is Fourier Transform

$$s(t) = \int_x \int_y m(x,y)\exp\left(-j2\pi\left(k_x(t)x + k_y(t)y\right)\right)dxdy$$
$$= M\left(k_x(t), k_y(t)\right)$$
$$= F\left[m(x,y)\right]\Big|_{k_x(t),k_y(t)}$$





K-space

At each point in time, the received signal is the Fourier transform of the object

$$s(t) = M\left(k_x(t), k_y(t)\right) = F\left[m(x,y)\right]\Big|_{k_x(t),k_y(t)}$$

evaluated at the spatial frequencies:

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

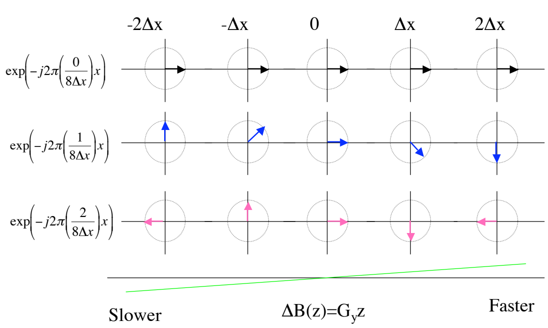
$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

Thus, the gradients control our position in k-space. The design of an MRI pulse sequence requires us to efficiently cover enough of k-space to form our image.

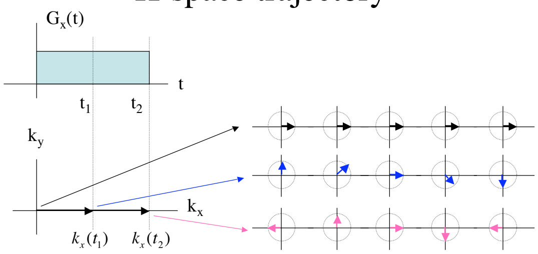
There is nothing that nuclear spins will not do for you, as long as you treat them as human beings.

Erwin Hahn

Interpretation



K-space trajectory



$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

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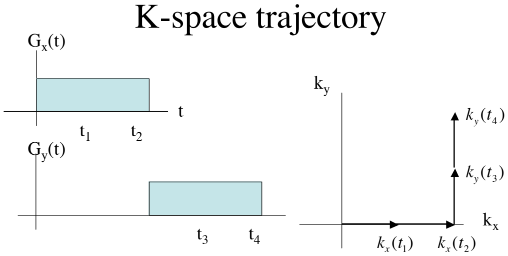
K-space trajectory



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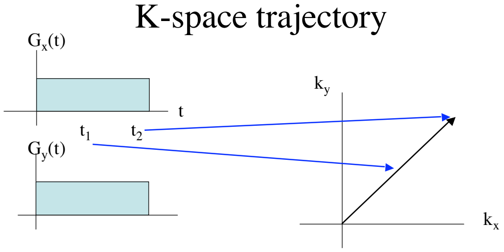
K-space trajectory



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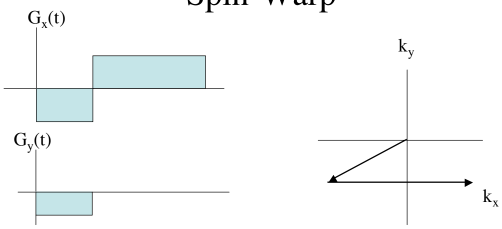
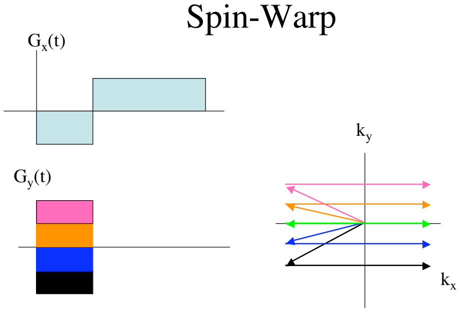
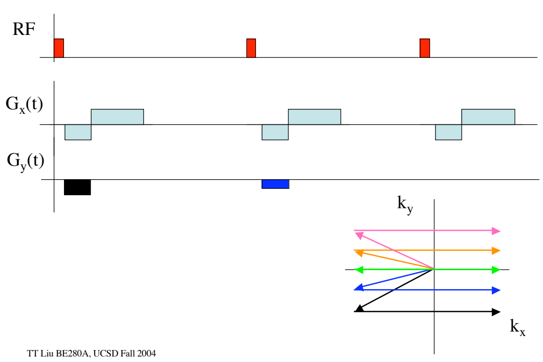
Spin-Warp



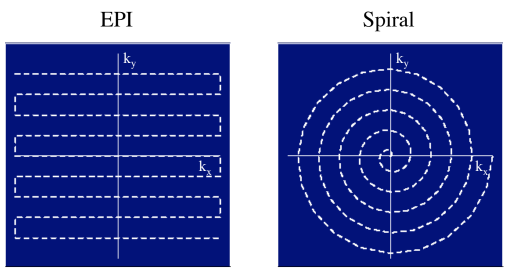
Spin-Warp



Spin-Warp Pulse Sequence



K-space trajectories



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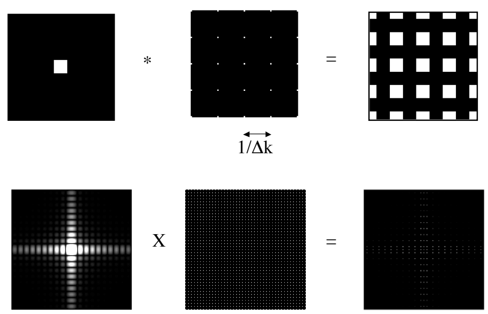
Credit: Larry Frank



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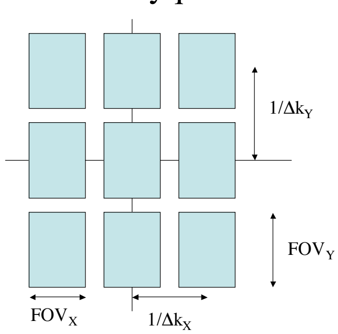
Nyquist Conditions



$1/\Delta k_Y > FOV_Y$

$1/\Delta k_X > FOV_X$

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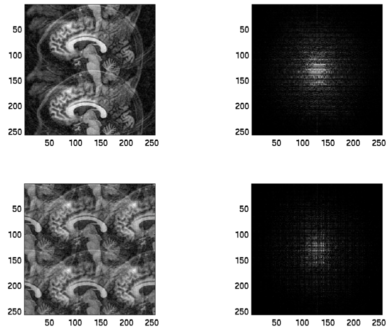
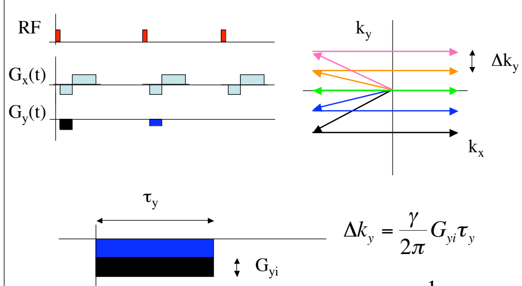
Aliasing



Sampling in k_y



$$\Delta k_y = \frac{\gamma}{2\pi} G_{yi} \tau_y$$

$$FOV_y = \frac{1}{\Delta k_y}$$

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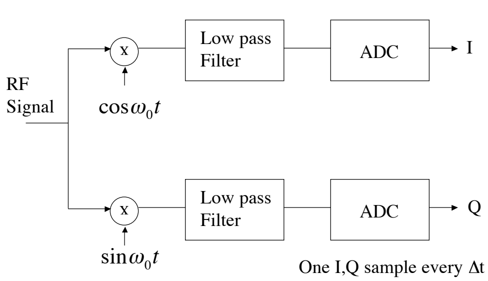
Sampling in k_x



One I,Q sample every Δt

M= I+jQ

Note: In practice, there are number of ways of implementing this processing.

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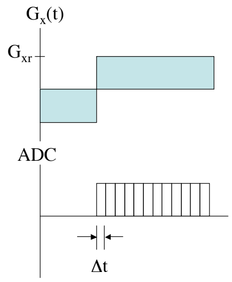
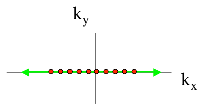
Sampling in k_x

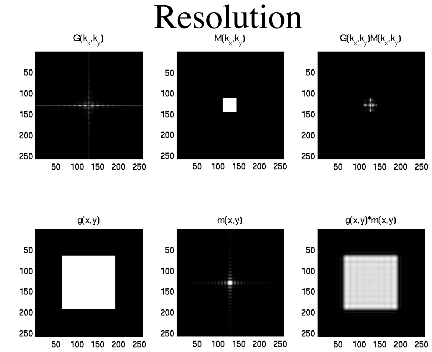




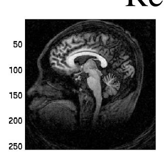
$$\Delta k_x = \frac{\gamma}{2\pi} G_{xr} \Delta t$$

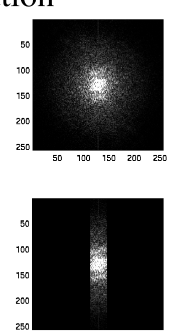
$$FOV_x = \frac{1}{\Delta k_x}$$

Resolution



Resolution



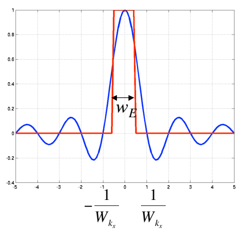


Effective Width

$$w_E = \frac{1}{w(0)}\int_{-\infty}^{\infty} w(x)dx$$

Example

$$\begin{aligned} w_E &= \frac{1}{1}\int_{-\infty}^{\infty} \mathrm{sinc}(W_{k_x}x)dx \\ &= F\left[\mathrm{sinc}(W_{k_x}x)\right]\Big|_{k_x=0} \\ &= \frac{1}{W_{k_x}} rect\left(\frac{k_x}{W_{k_x}}\right)\Bigg|_{k_x=0} \\ &= \frac{1}{W_{k_x}} \end{aligned}$$

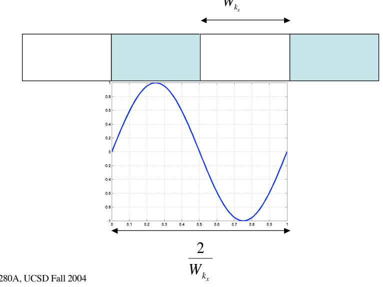


Resolution and spatial frequency

With a window of width W_{k_x} the highest spatial frequency is $W_{k_x}/2$.

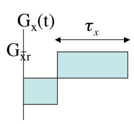
This corresponds to a spatial period of $2/W_{k_x}$.

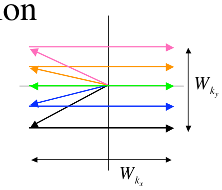
$\frac{1}{W_{k_x}}$ = Effective Width = δ_x = Resolution



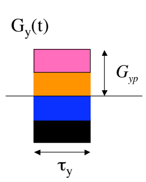
Resolution

$$\delta_x = \frac{1}{W_{k_x}} = \frac{1}{2k_{x,\max}} = \frac{1}{\frac{\gamma}{2\pi}G_{xr}\tau_x}$$





$$\delta_y = \frac{1}{W_{k_y}} = \frac{1}{2k_{y,\max}} = \frac{1}{\frac{\gamma}{2\pi}2G_{yp}\tau_y}$$



Example

Goal:

$FOV_x = FOV_y = 25.6$ cm

$\delta_x = \delta_y = 0.1$ cm

Readout Gradient:

$$FOV_x = \frac{1}{\frac{\gamma}{2\pi} G_{xr} \Delta t}$$

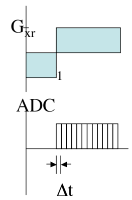
Pick $\Delta t = 32\ \mu$sec

$$G_{xr} = \frac{1}{FOV_x \frac{\gamma}{2\pi} \Delta t} = \frac{1}{(25.6cm)(42.57 \times 10^6 T^{-1}s^{-1})(32 \times 10^{-6} s)}$$

$$= 2.8675 \times 10^{-5} \text{ T/cm}$$

$$= .28675 \text{ G/cm}$$

1 Gauss = 1×10^{-4} Tesla



Example

Readout Gradient:

$$\delta_x = \frac{1}{\frac{\gamma}{2\pi} G_{xr} \tau_x}$$

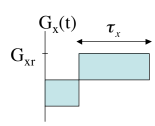
$$\tau_x = \frac{1}{\delta_x \frac{\gamma}{2\pi} G_{xr}} = \frac{1}{(0.1cm)(4257\ G^{-1}s^{-1})(0.28675 \text{ G/cm})}$$

$$= 8.192 \text{ ms}$$

$$= N_{read} \Delta t$$

where

$$N_{read} = \frac{FOV_x}{\delta_x} = 256$$



Example

Phase - Encode Gradient:

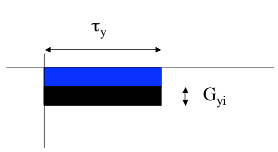
$$FOV_y = \frac{1}{\frac{\gamma}{2\pi} G_{yi} \tau_y}$$

Pick $\tau_y = 4.096\ m$sec

$$G_{yi} = \frac{1}{FOV_y \frac{\gamma}{2\pi} \tau_y} = \frac{1}{(25.6cm)(42.57 \times 10^6 T^{-1}s^{-1})(4.096 \times 10^{-3} s)}$$

$$= 2.2402 \times 10^{-7} \text{ T/cm}$$

$$= .00224 \text{ G/cm}$$



Example

Phase - Encode Gradient :

$$\delta_y = \frac{1}{\frac{\gamma}{2\pi} 2 G_{yp} \tau_y}$$

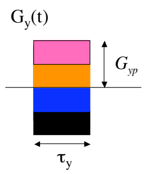
$$G_{yp} = \frac{1}{\delta_y 2 \frac{\gamma}{2\pi} \tau_y} = \frac{1}{(0.1cm)(4257\ G^{-1}s^{-1})(4.096 \times 10^{-3} s)}$$

$$= 0.2868 \text{ G/cm}$$

$$= \frac{N_p}{2} G_{yi}$$

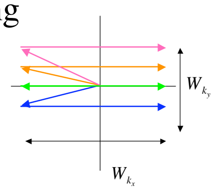
where

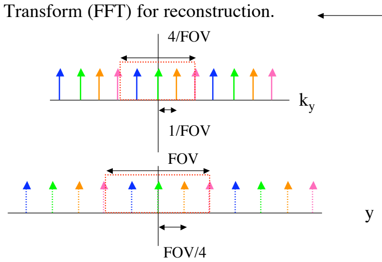
$$N_p = \frac{FOV_y}{\delta_y} = 256$$



Sampling

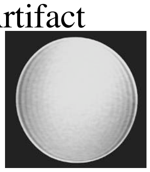
In practice, an even number (typically power of 2) sample is usually taken in each direction to take advantage of the Fast Fourier Transform (FFT) for reconstruction.



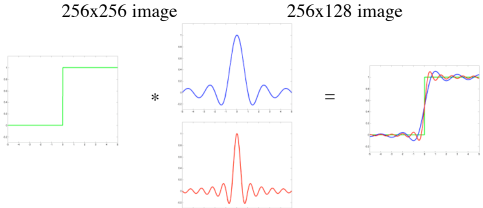


Gibbs Artifact

256x256 image

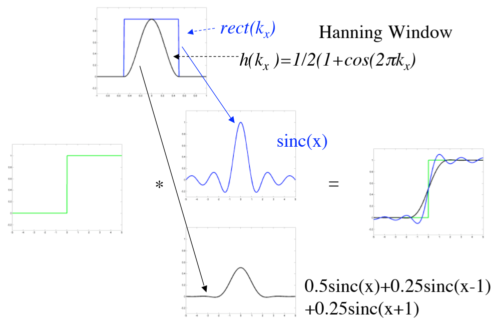


256x128 image



Images from http://www.mritutor.org/mritutor/gibbs.htm

Apodization



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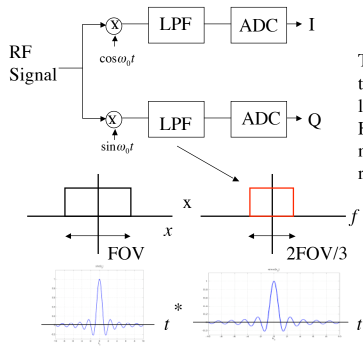
Images from http://www.mritutor.org/mritutor/gibbs.htm

Aliasing and Bandwidth



Temporal filtering in the readout direction limits the readout FOV. So there should never be aliasing in the readout direction.

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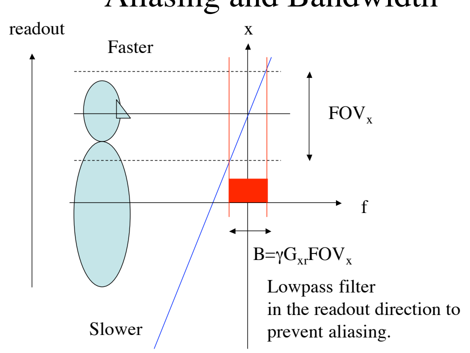
Aliasing and Bandwidth



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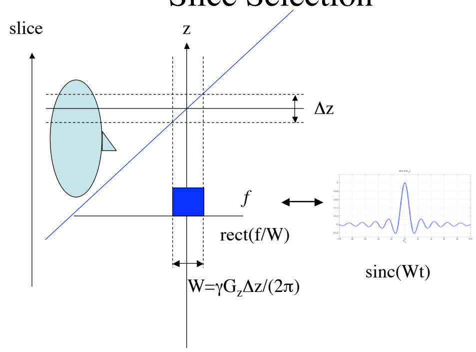
Slice Selection

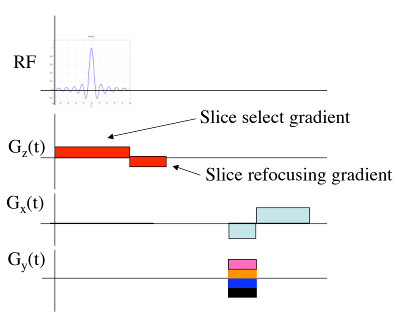
Recall, that we can tip spins away from their equilibrium state by applying a radio-frequency pulse at the Larmor frequency.

In the presence of a spatial gradient G_z, spins in an interval $-\Delta z/2$ to $-\Delta z/2$ have Larmor frequencies ranging from $\omega_0 - \gamma G_z \Delta z/2$ to $\omega_0 + \gamma G_z \Delta z/2$. In order to tip all the spins in this interval, we can apply an RF pulse with energy that is spaced over this frequency interval.

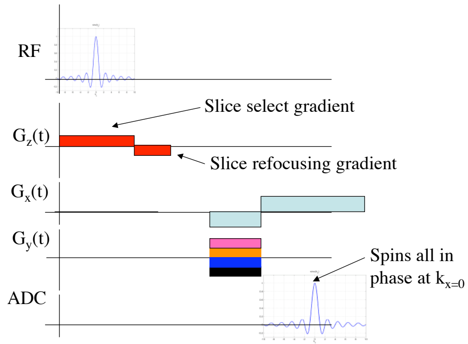
Slice Selection



Slice Selection



Gradient Echo



Static Inhomogeneities

In the ideal situation, the static magnetic field is totally uniform and the reconstructed object is determined solely by the applied gradient fields. In reality, the magnet is not perfect and will not be totally uniform. Part of this can be addressed by additional coils called "shim" coils, and the process of making the field more uniform is called "shimming". In the old days this was done manually, but modern magnets can do this automatically.

In addition to magnet imperfections, most biological samples are inhomogeneous and this will lead to inhomogeneity in the field. This is because, each tissue has different magnetic properties and will distort the field.

Static Inhomogeneities

The spatial nonuniformity in the field can be modeled by adding an additional term to our signal equation.

$$
\begin{aligned}
s_r(t) &= \int_V M(\vec{r},t)dV \\
&= \int_x \int_y \int_z M(x,y,z,0)e^{-t/T_2(\vec{r})}e^{-j\omega_0 t}e^{-j\omega_E(\vec{r})t} \exp\left(-j\gamma \int_o^t \vec{G}(\tau)\cdot\vec{r}\,d\tau\right)dxdydz
\end{aligned}
$$

The effect of this nonuniformity is to cause the spins to dephase with time and thus for the signal to decrease more rapidly. To first order this can be modeled as an additional decay term of the form

$$
s_r(t) = \int_x \int_y \int_z M(x,y,z,0)e^{-t/T_2(\vec{r})}e^{-t/T_2'(\vec{r})}e^{-j\omega_0 t} \exp\left(-j\gamma \int_o^t \vec{G}(\tau)\cdot\vec{r}\,d\tau\right)dxdydz
$$

T_2^* decay

The overall decay has the form.

$$\exp\left(-t/T_2^*(\bar{r})\right)$$

where

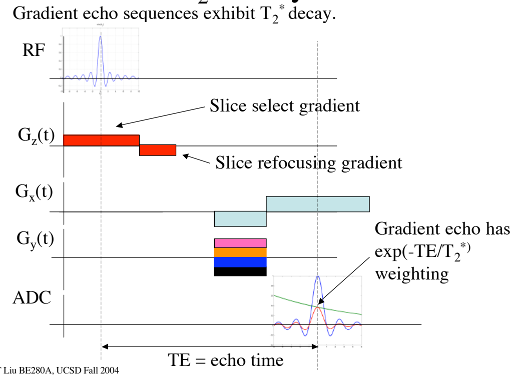
$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$

Due to random motions of spins. Not reversible.

Due to static inhomogeneities. Reversible with a spin-echo sequence.

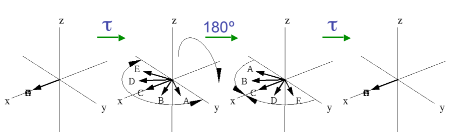
T_2^* decay

Gradient echo sequences exhibit T_2^* decay.



Spin Echo

Discovered by Erwin Hahn in 1950.

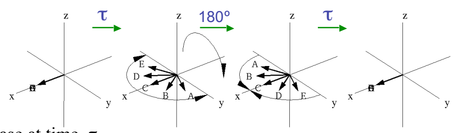


The spin-echo can refocus the dephasing of spins due to static inhomogeneities. However, there will still be T_2 dephasing due to random motion of spins.

There is nothing that nuclear spins will not do for you, as long as you treat them as human beings. Erwin Hahn

Image: Larry Frank

Spin Echo



Phase at time τ

$$\varphi(\tau) = \int_0^{\tau} -\omega_E(\vec{r})dt = -\omega_E(\vec{r})\tau$$

Phase after 180 pulse

$$\varphi(\tau^+) = \omega_E(\vec{r})\tau$$

Phase at time 2τ

$$\varphi(2\tau) = -\omega_E(\vec{r})\tau + \omega_E(\vec{r})\tau = 0$$

Image: Larry Frank

Spin Echo Pulse Sequence

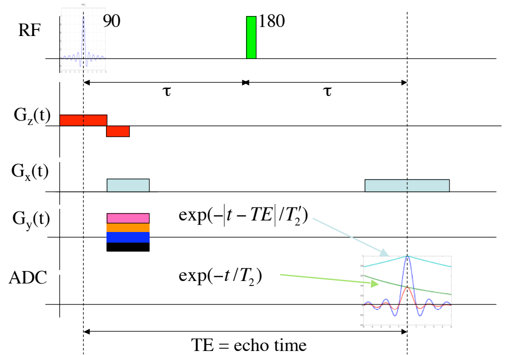
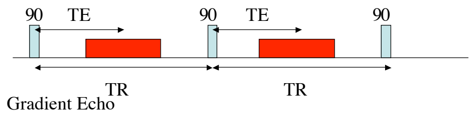


Image Contrast

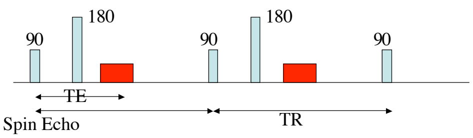
Different tissues exhibit different relaxation rates, T_1, T_2, and T_2^*. In addition different tissues can have different densities of protons. By adjusting the pulse sequence, we can create contrast between the tissues. The most basic way of creating contrast is adjusting the two sequence parameters: TE (echo time) and TR (repetition time).

Saturation Recovery Sequence



Gradient Echo

$$I(x,y) = \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right]e^{-TE/T_2^*(x,y)}$$



Spin Echo

$$I(x,y) = \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right]e^{-TE/T_2(x,y)}$$

T1-Weighted Scans

Make TE very short compared to either T_2 or T_2^*. The resultant image has both proton and T_1 weighting.

$$I(x,y) \approx \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right]$$

T2-Weighted Scans

Make TR very long compared to T_1 and use a spin-echo pulse sequence. The resultant image has both proton and T_2 weighting.

$$I(x,y) \approx \rho(x,y)e^{-TE/T_2}$$

Proton Density Weighted Scans

Make TR very long compared to T_1 and use a very short TE. The resultant image is proton density weighted.

$$I(x,y) \approx \rho(x,y)$$

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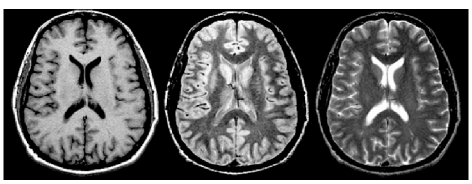
Example



T_1-weighted Density-weighted T_2-weighted

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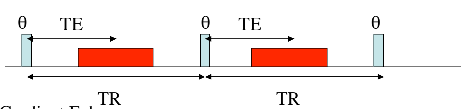
FLASH sequence



Gradient Echo

$$I(x,y) = \rho(x,y)\frac{\left[1-e^{-TR/T_1(x,y)}\right]\sin\theta}{\left[1-e^{-TR/T_1(x,y)}\cos\theta\right]}$$

Signal intensity is maximized at the Ernst Angle

$$\theta_E = \cos^{-1}\left(\exp(-TR/T_1)\right)$$

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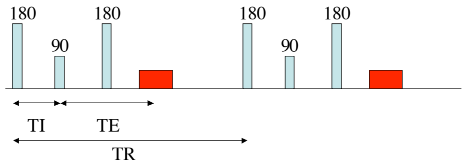
Inversion Recovery



$$I(x,y) = \rho(x,y)\left[1 - 2e^{-TI/T_1(x,y)} + e^{-TR/T_1(x,y)}\right]e^{-TE/T_2(x,y)}$$

Intensity is zero when inversion time is

$$TI = -T_1 \ln\left[\frac{1+\exp(-TR/T_1)}{2}\right]$$