Computing Transforms

$$F(1) = \int_{-\infty}^{\infty} e^{-j2\pi k_x x} dx = ???$$

Define $h(k_x) = \int_{-\infty}^{\infty} e^{-j2\pi k_x x} dx$ and see what it does under an integral.

$$\begin{aligned}
\int_{-\infty}^{\infty} G(k_x) h(k_x) dk_x &= \int_{-\infty}^{\infty} G(k_x) \int_{-\infty}^{\infty} e^{-j2\pi k_x x} dx dk_x \\
&= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(k_x) e^{-j2\pi k_x x} dk_x dx \\
&= \int_{-\infty}^{\infty} g(-x) dx \\
&= G(0)
\end{aligned}$$

Therefore, $F(1) = \int_{-\infty}^{\infty} e^{-j2\pi k_x x} dx = \delta(k_x)$

Application of Duality

$$F\{\text{sinc}(x)\} = \int_{-\infty}^{\infty} \frac{\sin \pi x}{\pi x} e^{-j2\pi k_x x} dx = ??$$

Recall that $F\{\Pi(x)\} = \text{sinc}(k_x)$.

Therefore from duality, $F\{\text{sinc}(x)\} = \Pi(-k_x) = \Pi(k_x)$

Application of Convolution Thm.

$$\Lambda(x) = \Pi(x) * \Pi(x)$$

$$F(\Lambda(x)) = \operatorname{sinc}^2(k_x)$$