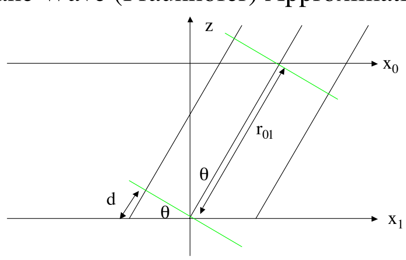
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2005
Ultrasound Lecture 2

Plane Wave (Fraunhofer) Approximation



$$d = -x_1 \sin\theta$$
$$\sin\theta = \frac{x_0}{r_{01}} \approx \frac{x_0}{z}$$
$$d \approx -\frac{x_0 x_1}{z}$$

Plane Wave Approximation

$$\frac{1}{r}\exp(jkr) \approx \frac{1}{z}\exp\big(jk(z+d)\big) = \frac{1}{z}\exp\left(j\frac{2\pi}{\lambda}\left(z - \frac{x_0 x_1}{z}\right)\right)$$
$$U(x_0) = \int_{-\infty}^{\infty} s(x_1)\frac{1}{r}\exp(jkr)dx_1$$
$$\approx \int_{-\infty}^{\infty} s(x_1)\frac{1}{z}\exp\left(j\frac{2\pi}{\lambda}\left(z - \frac{x_0 x_1}{z}\right)\right)dx_1$$
$$= \frac{1}{z}\exp\left(j\frac{2\pi z}{\lambda}\right)\int_{-\infty}^{\infty} s(x_1)\exp\left(-\frac{j2\pi x_0 x_1}{\lambda z}\right)dx_1$$
$$= \frac{1}{z}\exp\left(j\frac{2\pi z}{\lambda}\right)\int_{-\infty}^{\infty} s(x_1)\exp\left(-j2\pi k_x x_1\right)dx_1$$
$$= \frac{1}{z}\exp\left(j\frac{2\pi z}{\lambda}\right)F\big[s(x)\big]\Big|_{k_x = \frac{x_0}{\lambda z}}$$

Plane Wave Approximation

In general

$$U(x_0,y_0)=\frac{1}{z}\exp\left(j\frac{2\pi z}{\lambda}\right)F\left[s(x,y)\right]\Big|_{k_x=\frac{x_0}{\lambda z},k_y=\frac{y_0}{\lambda z}}$$

Example

$$s(x,y)=rect(x/D)rect(y/D)$$

$$U(x_0,y_0)=\frac{1}{z}\exp(jkz)D^2\,\mathrm{sinc}\left(Dk_x\right)\mathrm{sinc}\left(Dk_y\right)$$

$$=\frac{1}{z}\exp(jkz)D^2\,\mathrm{sinc}\left(D\frac{x_0}{\lambda z}\right)\mathrm{sinc}\left(Dk_y\frac{y_0}{\lambda z}\right)$$

Zeros occur at $x_0=\frac{n\lambda z}{D}$ and $y_0=\frac{n\lambda z}{D}$

Beamwidth of the sinc function is $\frac{\lambda z}{D}$

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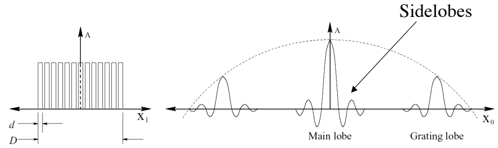
Example



$$rect\left(\frac{x}{D}\right)\left[rect\left(\frac{x}{d}\right)*\frac{1}{d}comb\left(\frac{x}{d}\right)\right]\Leftrightarrow D\,\mathrm{sinc}(Dk_x)*\left[d\,\mathrm{sinc}(dk_x)comb(dk_x)\right]$$

Question: What should we do to reduce the sidelobes?

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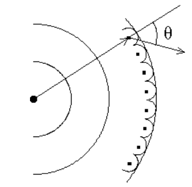
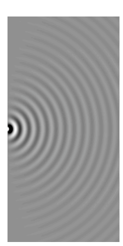
Huygen's Principle



http://www.fink.com/thesis/chapter2.html



http://www.cbem.imperial.ac.uk/ardan/diff/hfw.html

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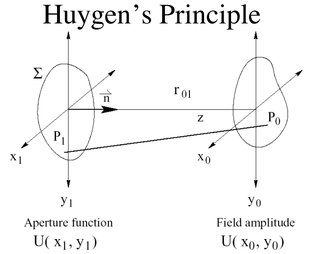
Huygen's Principle



Wavenumber

$$k = \frac{2\pi}{\lambda}$$

$$U(P_0) = \iint_\Sigma h(P_0, P_1)U(P_1)ds$$

$$\text{where } h(P_0, P_1) = \frac{1}{j\lambda}\frac{\exp(jkr_{01})}{r_{01}}\cos(\overrightarrow{n}, \overrightarrow{r}_{01})$$

Oliquity Factor

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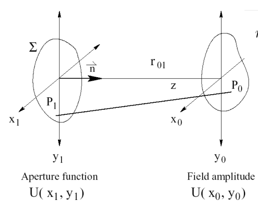
Small-Angle (paraxial) Approximation



$$r_{01} = \sqrt{z^2 + (x_1 - x_0)^2 + (y_1 - y_0)^2}$$

$$cos(\overrightarrow{n}, \overrightarrow{r}_{01}) \approx 1$$

$$r_{01} \approx z$$

$$h(x_0, y_0; x_1, y_1) \approx \frac{1}{j\lambda z}\exp(jkr_{01})$$

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Fresnel Approximation

$$\begin{aligned} r_{01} &= \sqrt{z^2 + (x_1 - x_0)^2 + (y_1 - y_0)^2} \\ &= z\sqrt{1 + \left(\frac{(x_1 - x_0)}{z}\right)^2 + \left(\frac{(y_1 - y_0)^2}{z}\right)} \\ &\approx z\left[1 + \frac{1}{2}\left(\frac{(x_1 - x_0)}{z}\right)^2 + \frac{1}{2}\left(\frac{(y_1 - y_0)}{z}\right)^2\right] \end{aligned}$$

Approximates spherical wavefront with a parabolic phase profile

$$h(x_0, y_0; x_1, y_1) \approx \frac{\exp(jkz)}{j\lambda z}\exp\left[\frac{jk}{2z}\left[(x_1 - x_0)^2 + (y_1 - y_0)^2\right]\right]$$

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Fresnel Approximation

$$U(x_0,y_0)=\iint \frac{\exp(jkz)}{j\lambda z}\exp\left(\frac{jk}{2z}\left((x_1-x_0)^2+(y_1-y_0)^2\right)\right)s(x_1,y_1)dx_1dy_1$$
$$=\frac{\exp(jkz)}{j\lambda z}\left(s(x_0,y_0)**\exp\left(\frac{jk}{2z}\left(x_0^2+y_0^2\right)\right)\right)$$

Fraunhofer Approximation

$$kr_{01}\approx kz\left(1+\frac{1}{2}\left(\frac{x_1-x_0}{z}\right)^2+\frac{1}{2}\left(\frac{y_1-y_0}{z}\right)^2\right)$$
$$=kz\left(1+\frac{1}{2z^2}\left(x_1^2-2x_1x_0+x_0^2\right)+\frac{1}{2z^2}\left(y_1^2-2y_1y_0+y_0^2\right)\right)$$
$$=kz+\frac{k}{2z}\left(x_1^2+y_1^2\right)+\frac{k}{2z}\left(x_0^2+y_0^2\right)-\frac{k}{z}\left(x_1x_0+y_1y_0\right)$$
$$\approx kz++\frac{k}{2z}\left(x_0^2+y_0^2\right)-\frac{k}{z}\left(x_1x_0+y_1y_0\right)$$

Assume this term is negligible.

Fraunhofer Condition

Phase term due to position on transducer is $\frac{k}{2z}\left(x_1^2+y_1^2\right)$

Far-field condition is

$$\frac{k}{2z}\left(x_1^2+y_1^2\right)<<1$$
$$z>>\frac{k}{2}\left(x_1^2+y_1^2\right)=\frac{\pi}{\lambda}\left(x_1^2+y_1^2\right)$$

For a square DxD transducer, $x_1^2+y_1^2=D^2/4$

$$z>>\frac{\pi D^2}{4\lambda}\approx\frac{D^2}{\lambda}$$

Fraunhofer Approximation

$$U(x_0,y_0) \approx \frac{\exp(jkz)\exp\left[\frac{jk}{2z}(x_0^2+y_0^2)\right]}{j\lambda z}\iint_{-\infty}^{\infty} U(x_1,y_1)\exp\left[-\frac{j2\pi}{\lambda z}(x_0x_1+y_0y_1)\right]dx_1dy_1$$

Quadratic phase term

Fourier transform of the source with

$$k_x = \frac{x_0}{\lambda z} \quad k_y = \frac{y_0}{\lambda z}$$

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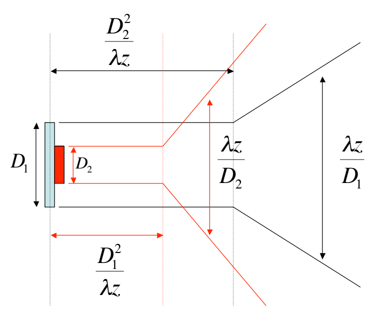

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Transducer Dimension

Goal : Operate in the Fresnel Zone

$z < D^2/\lambda$

$D_{opt} \approx \sqrt{\lambda z_{\max}}$

Example

$z_{\max} = 20\ cm$

$\lambda = 0.5\ mm$

$D_{opt} = 1\ cm$

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Focusing in Fresnel Zone

$$
\begin{aligned}
U(x_0,y_0) &= \iint \frac{\exp(jkz)}{j\lambda z}\exp\left(\frac{jk}{2z}\left((x_1-x_0)^2+(y_1-y_0)^2\right)\right)s(x_1,y_1)dx_1dy_1 \\
&= \iint \frac{\exp(jkz)}{j\lambda z}\exp\left(\frac{jk}{2z}\left((x_1^2+y_1^2)+(x_0^2+y_0^2)-2(x_1x_0+y_1y_0)\right)\right)s(x_1,y_1)dx_1dy_1 \\
&= \frac{\exp(jkz)}{j\lambda z}\exp\left(\frac{jk}{2z}(x_0^2+y_0^2)\right)\iint \frac{\exp(jkz)}{j\lambda z}\exp\left(\frac{jk}{2z}(x_1^2+y_1^2)\right)\exp\left(-\frac{jk}{z}(x_1x_0+y_1y_0)\right)s(x_1,y_1)dx_1dy_1
\end{aligned}
$$

Use time delays to compensate for this phase term

$$U(x_0,y_0) = \frac{\exp(jkz)}{j\lambda z}\exp\left(\frac{jk}{2z}(x_0^2+y_0^2)\right)F\left[\exp\left(\frac{jk}{2z}(x_1^2+y_1^2)\right)s(x_1,y_1)\right]$$

Focusing in Fresnel Zone

$$U(x_0,y_0) = \frac{\exp(jkz)}{j\lambda z}\exp\left(\frac{jk}{2z}(x_0^2+y_0^2)\right)F\left[\exp\left(\frac{jk}{2z}(x_1^2+y_1^2)\right)s(x_1,y_1)\right]$$

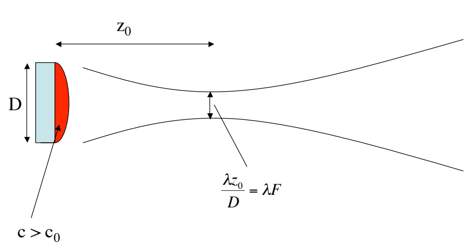
Make $s(x_1,y_1) = s_0(x_1,y_1)\exp\left(-\frac{jk}{2z_0}(x_1^2+y_1^2)\right)$

At the focal depth $z = z_0$

$$U(x_0,y_0) = \frac{\exp(jkz_0)}{j\lambda z_0}\exp\left(\frac{jk}{2z_0}(x_0^2+y_0^2)\right)F\left[s(x_1,y_1)\right]$$

Beamwidth at the focal depth is: $\frac{\lambda z_0}{D}$

Acoustic Lens



Depth of Focus

When $z \neq z_0$, the phase term is $\Delta\Phi = \exp\left(-\frac{jk}{2z_0}\left(x_1^2 + y_1^2\right)\right)\exp\left(-\frac{jk}{2z}\left(x_1^2 + y_1^2\right)\right)$

and the lens is not perfectly focused.

Consider variation in the x - direction.

$$\Delta\Phi = \frac{kx^2}{2}\left(\frac{1}{z} - \frac{1}{z_0}\right)$$

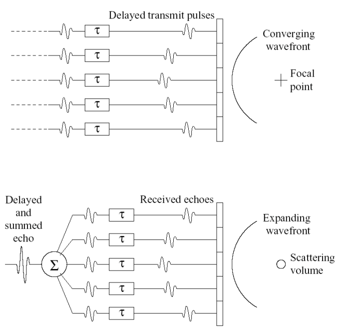
For transducer of size D, $\frac{x^2}{2} = \frac{D^2}{4}$

If we want $|\Delta\Phi| = \left|\frac{\pi D^2}{2\lambda}\left(\frac{1}{z} - \frac{1}{z_0}\right)\right| < 1$ radian then

$$\left|\frac{1}{z} - \frac{1}{z_0}\right| < \frac{2\lambda}{\pi D^2}$$

The larger the D, the smaller the depth of focus.

Focusing with Phased Array



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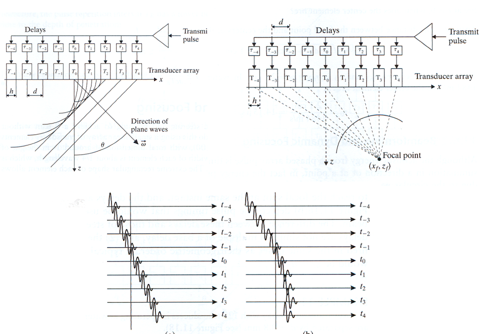
Focusing and Steering



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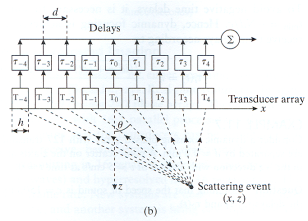
Dynamic Focusing



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