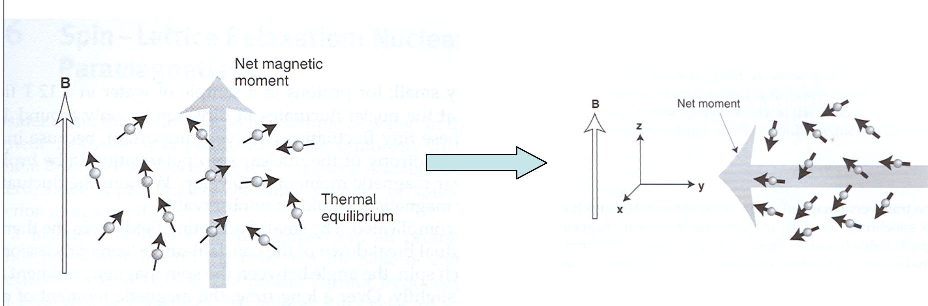
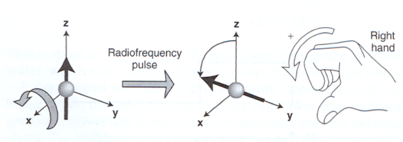
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2006
MRI Lecture 4

Thomas Liu, BE280A, UCSD, Fall 2006

RF Excitation





From Levitt, Spin Dynamics, 2001

Thomas Liu, BE280A, UCSD, Fall 2006

RF Excitation

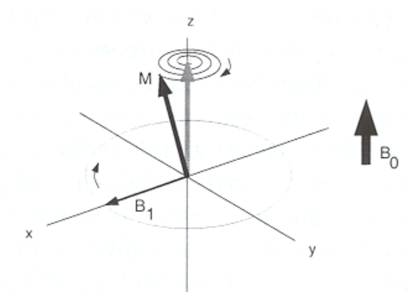


Image & caption: Nishimura, Fig. 3.2

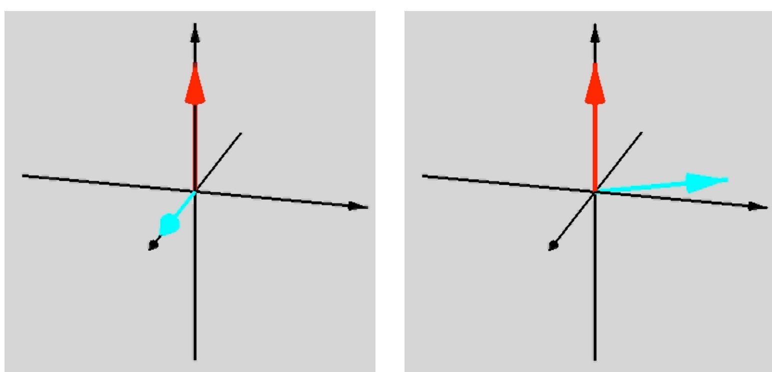
At equilibrium, net magnetizaion is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

B_1 radiofrequency field tuned to Larmor frequency and applied in transverse (*xy*) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the *z*-axis.

- lab frame of reference

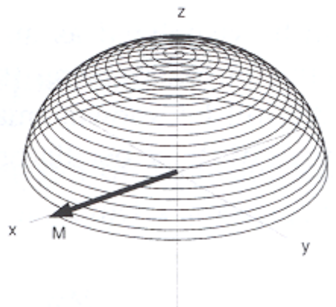
Thomas Liu, BE280A, UCSD, Fall 2006

RF Excitation



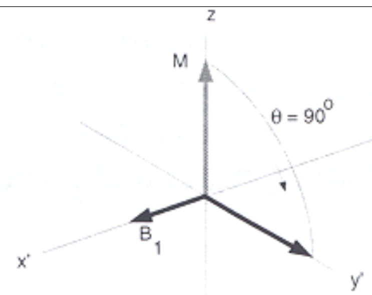
Thomas Liu, BE280A, UCSD, Fall 2006

http://www.eecs.umich.edu/%7Ednoll/BME516/

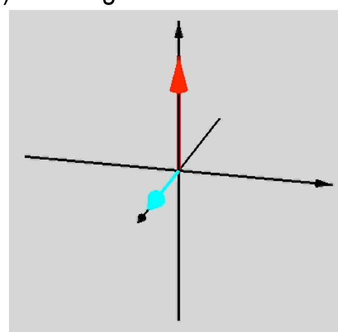


a) Laboratory frame behavior of **M**

Images & caption: Nishimura, Fig. 3.3

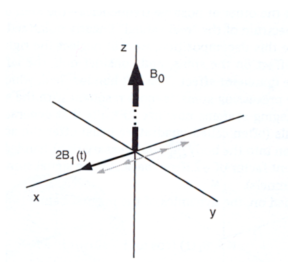


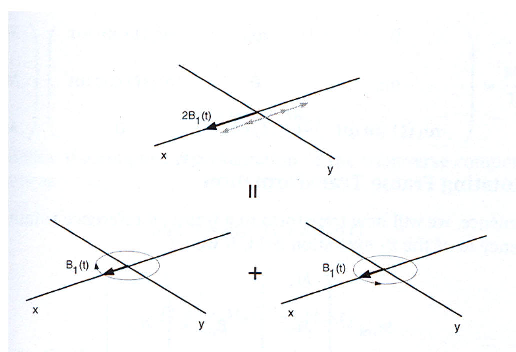
b) Rotating frame behavior of **M**



Thomas Liu, BE280A, UCSD, Fall 2006

http://www.eecs.umich.edu/%7Ednoll/BME516/





$$\mathbf{B_1}(t) = 2B_1(t)\cos(\omega t)\mathbf{i}$$
$$= B_1(t)\big(\cos(\omega t)\mathbf{i} - \sin(\omega t)\mathbf{j}\big) + B_1(t)\big(\cos(\omega t)\mathbf{i} + \sin(\omega t)\mathbf{j}\big)$$

Thomas Liu, BE280A, UCSD, Fall 2006

Nishimura 1996

Rotating Frame Bloch Equation

$$\frac{d\mathbf{M}_{rot}}{dt} = \mathbf{M}_{rot} \times \gamma \mathbf{B}_{eff}$$

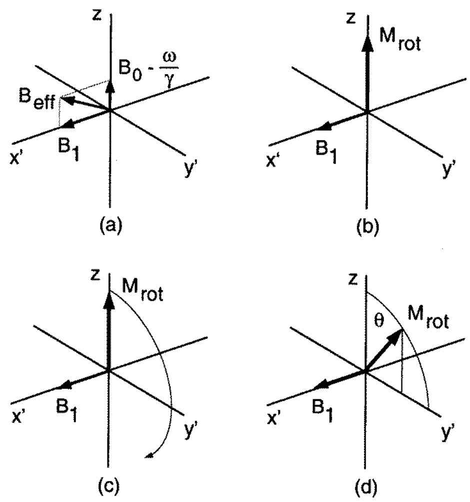
$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}; \quad \omega_{rot} = \begin{bmatrix} 0 \\ 0 \\ -\omega \end{bmatrix}$$

$$\textit{Let } \mathbf{B}_{rot} = B_1(t)\mathbf{i} + B_0\mathbf{k}$$

$$\begin{aligned}\mathbf{B}_{eff} &= \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma} \\ &= B_1(t)\mathbf{i} + \left(B_0 - \frac{\omega}{\gamma}\right)\mathbf{k}\end{aligned}$$

$$\begin{aligned}\text{If } \omega &= \omega_0 \\ &= \gamma B_0\end{aligned}$$

$$\text{Then } \mathbf{B}_{eff} = B_1(t)\mathbf{i}$$



Flip angle

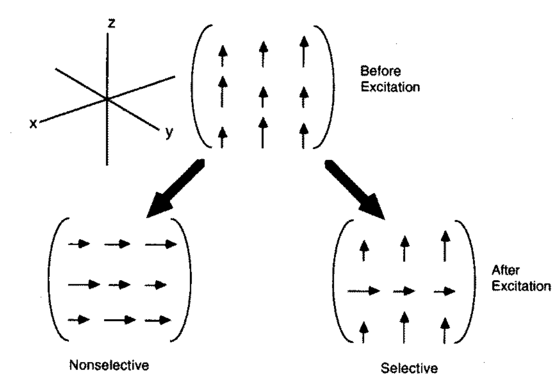
$$\theta = \int_0^{\tau} \omega_1(s)ds$$

where

$$\omega_1(t) = \gamma B_1(t)$$

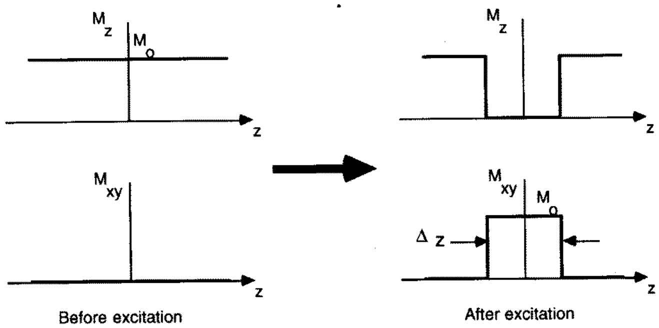
Thomas Liu, BE280A, UCSD, Fall 2006

Nishimura 1996



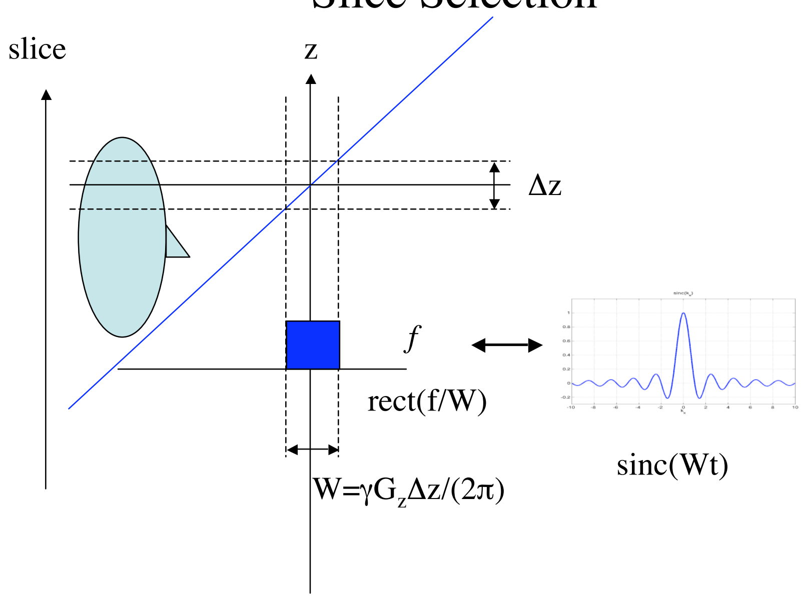
Thomas Liu, BE280A, UCSD, Fall 2006

Nishimura 1996



Nishimura 1996

Slice Selection



$$Let\ \mathbf{B}_{rot} = B_1(t)\mathbf{i} + \left(B_0 + \gamma G_z z\right)\mathbf{k}$$

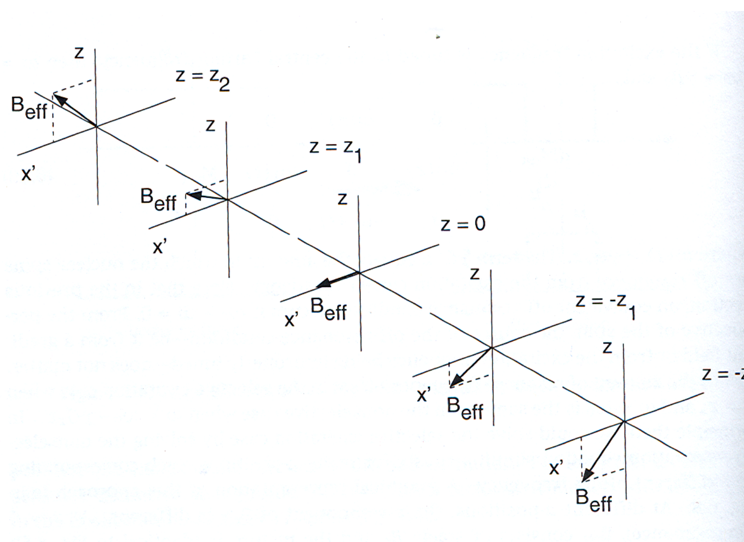
$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}$$

$$= B_1(t)\mathbf{i} + \left(B_0 + \gamma G_z z - \frac{\omega}{\gamma}\right)\mathbf{k}$$

If $\omega = \omega_0$

$$\mathbf{B}_{eff} = B_1(t)\mathbf{i} + \left(\gamma G_z z\right)\mathbf{k}$$

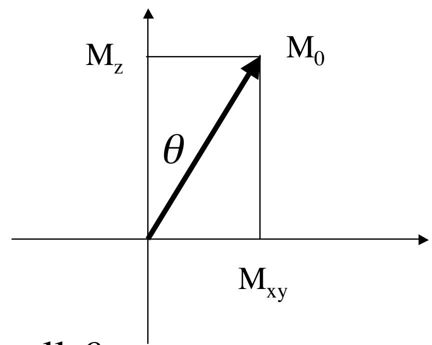
Thomas Liu, BE280A, UCSD, Fall 2006



Thomas Liu, BE280A, UCSD, Fall 2006

Nishimura 1996

Small Tip Angle Approximation



For small θ

$$M_z = M_0 \cos\theta \approx M_0$$

$$M_{xy} = M_0 \sin\theta \approx M_0\theta$$

Recall that in the rotating frame, flip angle $\theta = \gamma \int_0^{\tau} B_1(s)ds$

Define $\omega(z)$ as the Larmor Frequency at each location z referenced to ω_0

The effective field felt in each spin's rotating frame of reference is:

$$B_1^e(t) = B_1(t)\exp(j\omega(z)t)$$

Therefore the flip angle in each spin's frame of reference is

$$\theta(t,z) = \gamma \int_0^t \exp(j\omega(z)s)B_1(s)ds$$

With respect to the on-resonance frame of reference, there is also a relative phase shift of $\exp(-j\omega(z)t)$, so that

$$\theta(t,z) = \gamma \exp(-j\omega(z)t)\int_0^t \exp(j\omega(z)s)B_1(s)ds$$

Applying small angle approximation leads to

$$M_r(t,z) \approx \mathrm{j}\mathrm{M}_0\theta(t,z) = \mathrm{M}_0\gamma \exp(-j\omega(z)t)\int_0^t \exp(j\omega(z)s)B_1(s)ds$$

Small Tip Angle Approximation

$$\mathrm{M_r}(t,z) = jM_0 \exp(-j\omega(z)t)\int_0^t \exp(j\omega(z)s)\omega_1(s)ds$$

For symmetric pulse of length τ

$$\mathrm{M_r}(\tau,z) = jM_0 \exp(-j\omega(z)\tau/2)\int_{-\tau/2}^{\tau/2} \exp(j2\pi f(z)s)\omega_1(s+\tau/2)ds$$

$$= jM_0 \exp(-j\omega(z)\tau/2)F\left\{\omega_1(t+\tau/2)\right\}\Big|_{f=-f(z)=-\frac{\gamma}{2\pi}G_z z}$$

Small Tip Angle Example

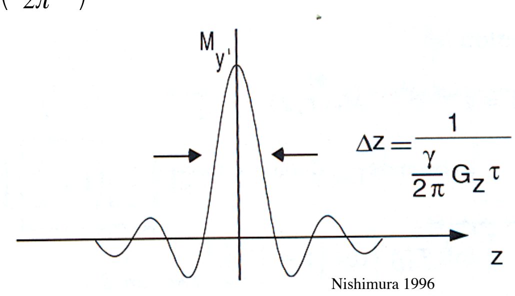
$$B_1(t) = B_1 rect\left(\frac{t-\tau/2}{\tau}\right)$$

$$M_r(\tau,z) = jM_0 \exp(-j\omega(z)\tau)\int_0^\tau \exp(j\omega(z)s)\omega_1 rect\left(\frac{s-\tau/2}{\tau}\right)ds$$

$$= jM_0 \exp(-j\omega(z)\tau/2)F_{1D}\left(\omega_1 rect\left(\frac{t}{\tau}\right)\right)\Big|_{f=-(\gamma/2\pi)G_z z}$$

$$= jM_0 \exp(-j\omega(z)\tau/2)\omega_1\tau \sin c(f\tau)$$

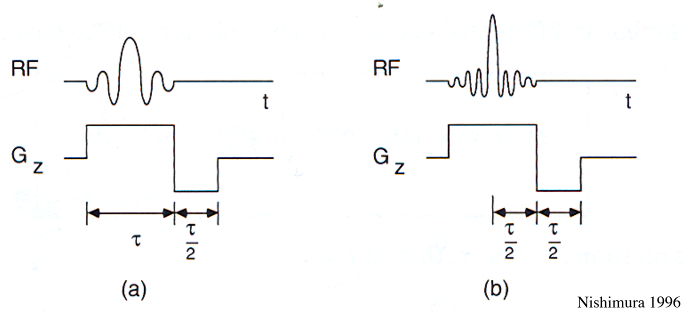
$$= jM_0 \exp(-j\omega(z)\tau/2)\omega_1\tau \sin c\left(\frac{\gamma G_z\tau}{2\pi}z\right)$$



Nishimura 1996

Refocusing

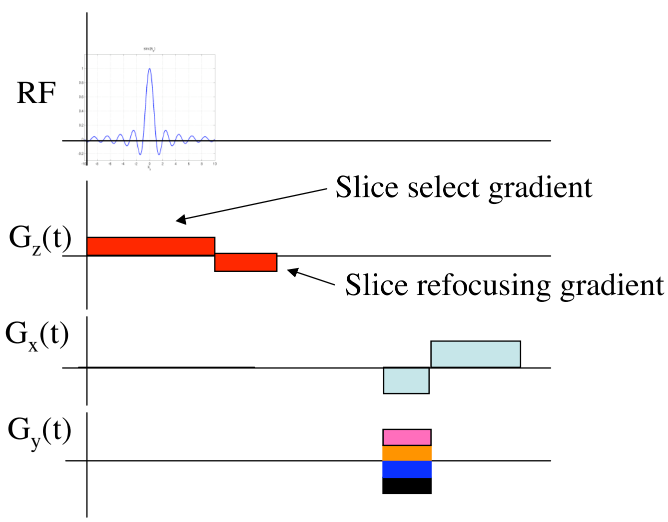
$$M_r(3\tau/2,z) = \exp\left(j\omega(z)\tau/2\right)M_r(\tau,z)$$
$$= jM_0\exp\left(j\omega(z)\tau/2\right)\exp\left(-j\omega(z)\tau/2\right)F\{\omega_1(t+\tau/2)\}\Big|_{f=-\frac{\gamma}{2\pi}G_z z}$$
$$= jM_0F\{\omega_1(t+\tau/2)\}\Big|_{f=-\frac{\gamma}{2\pi}G_z z}$$



Nishimura 1996

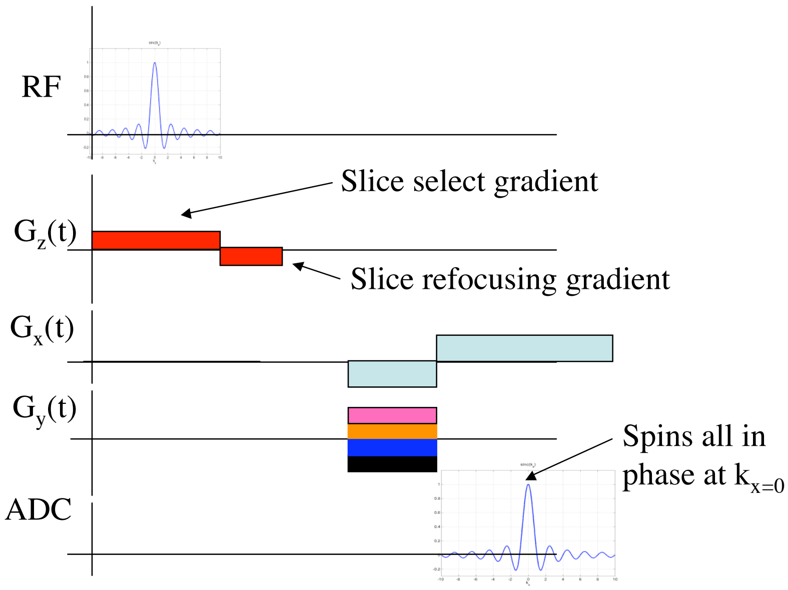
Thomas Liu,

Slice Selection



Thomas Liu, BE280A, UCSD, Fall 2006

Gradient Echo



Thomas Liu, BE280A, UCSD, Fall 2006

Small Tip Angle Example

$$B_1(t+\tau/2) = A\,\mathrm{sinc}\left(t/\tau\right)\left(0.5+0.46\cos\left(\frac{2\pi t}{\tau}\right)\right)$$

$$= A\,\mathrm{sinc}\left(t/\tau\right)w(t)$$

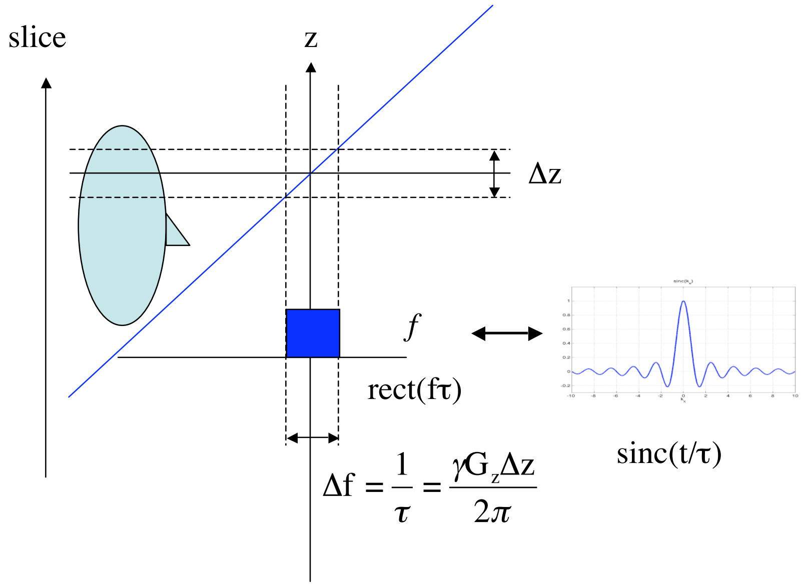
$$F(B_1(t+\tau/2)) = A\tau\;\mathrm{rect}(f\tau)*W(f)$$

$$= A\tau\;\mathrm{rect}\left(\frac{\gamma G_z z\tau}{2\pi}\right)*W\left(-\frac{\gamma G_z z}{2\pi}\right)$$

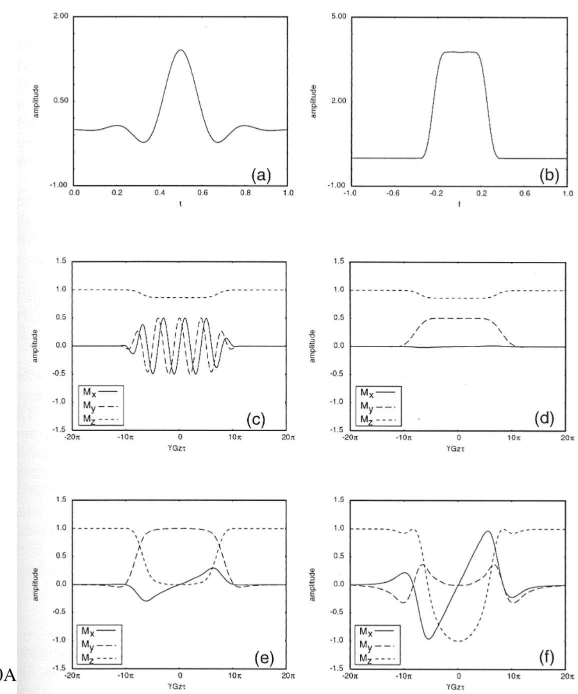
Width of the rect function is $\Delta z = \dfrac{2\pi}{\gamma G_z \tau}$

Thomas Liu, BE280A, UCSD, Fall 2006

Slice Selection



$$\Delta f = \frac{1}{\tau} = \frac{\gamma G_z \Delta z}{2\pi}$$



Nishimura 1996

Example

$\Delta z = 5\ mm; \tau = 400\ \mu\text{sec};\ \theta = \pi/2$

$$G_z = \frac{2\pi}{\gamma \Delta z \tau} = \frac{1}{(4257 Hz/G)(0.5cm)(400e-6)} = 1.175\ G/cm$$

$$\theta \approx \gamma \int_0^T B_1 \sin c\left(\frac{s - T/2}{\tau}\right) ds \approx \gamma B_1 \cdot \left(\text{area of sinc}\right) = \gamma B_1 \tau$$

$$B_1 = \frac{\theta}{\gamma \tau} = \frac{\pi/2}{2\pi(4257 Hz/G)(400e-6)} = 0.1468\ G$$