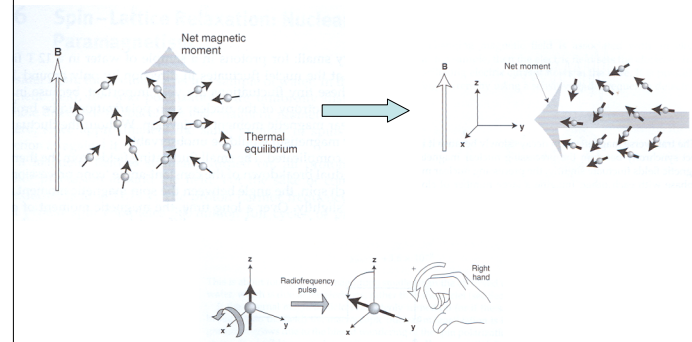


Bioengineering 280A
Principles of Biomedical Imaging
Fall Quarter 2007
MRI Lecture 4

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RF Excitation



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From Levitt, Spin Dynamics, 2001

RF Excitation

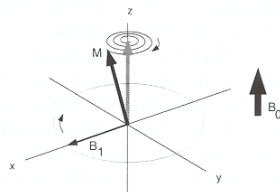


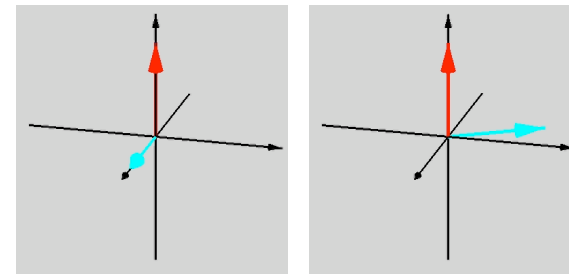
Image & caption: Nishimura, Fig. 3.2

At equilibrium, net magnetization is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

B_1 radiofrequency field tuned to Larmor frequency and applied in transverse (xy) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the z -axis.
- lab frame of reference

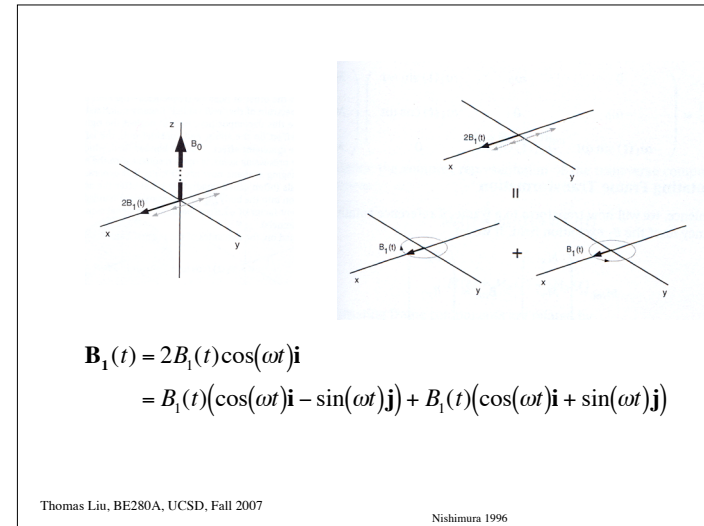
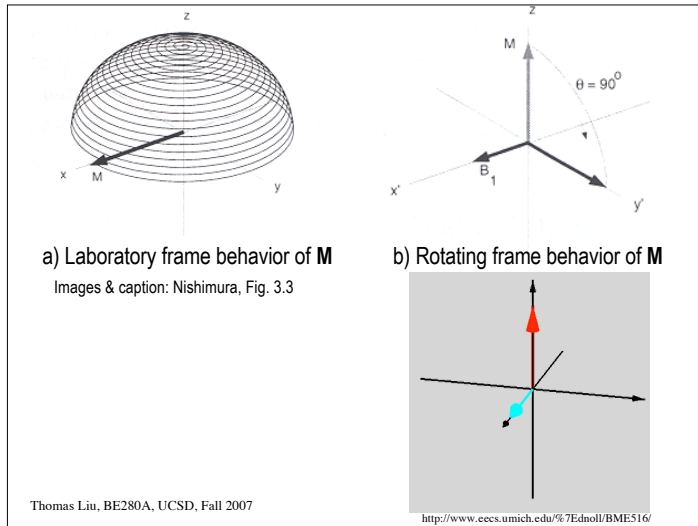
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<http://www.eecs.umich.edu/%7Ednoll/BME516/>



Rotating Frame Bloch Equation

$$\frac{d\mathbf{M}_{rot}}{dt} = \mathbf{M}_{rot} \times \gamma \mathbf{B}_{eff}$$

$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma} \mathbf{k}; \quad \omega_{rot} = \begin{bmatrix} 0 \\ 0 \\ -\omega \end{bmatrix}$$

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Let $\mathbf{B}_{rot} = B_1(t)\mathbf{i} + B_0\mathbf{k}$

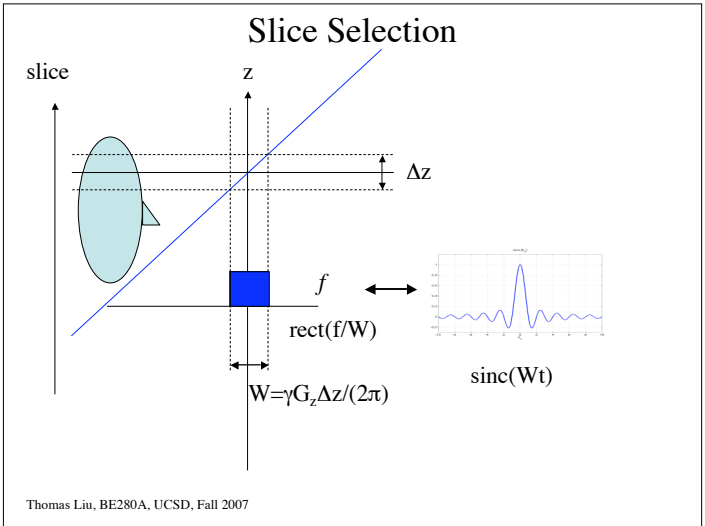
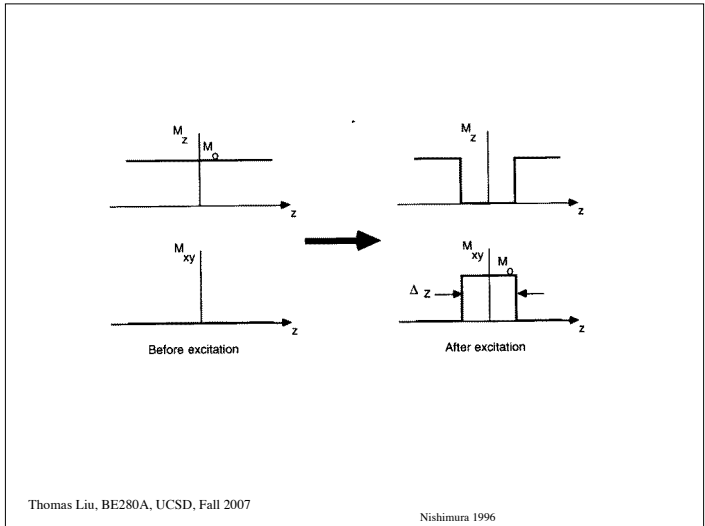
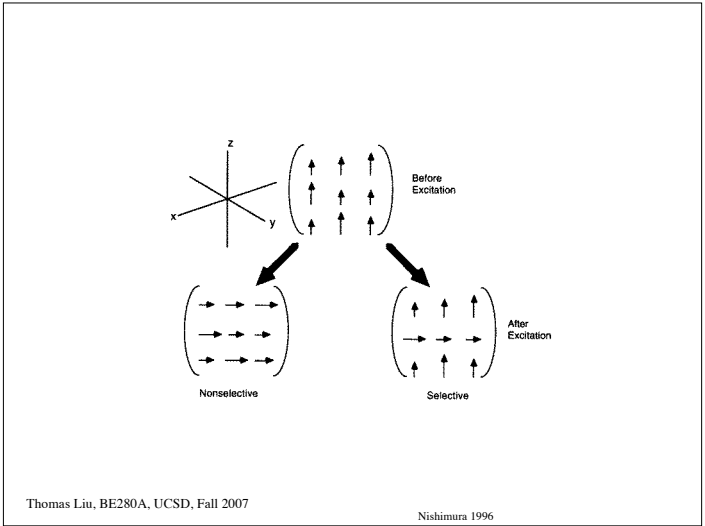
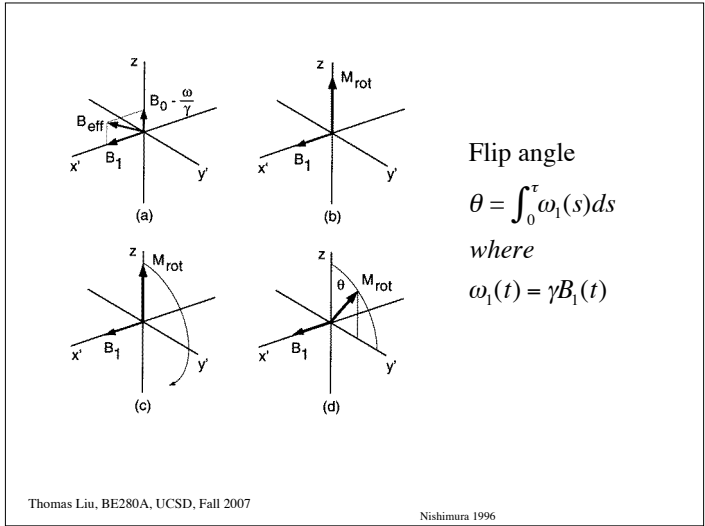
$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma} \mathbf{k}$$

$$= B_1(t)\mathbf{i} + \left(B_0 - \frac{\omega}{\gamma}\right)\mathbf{k}$$

If $\omega = \omega_0$
 $= \gamma B_0$

Then $\mathbf{B}_{eff} = B_1(t)\mathbf{i}$

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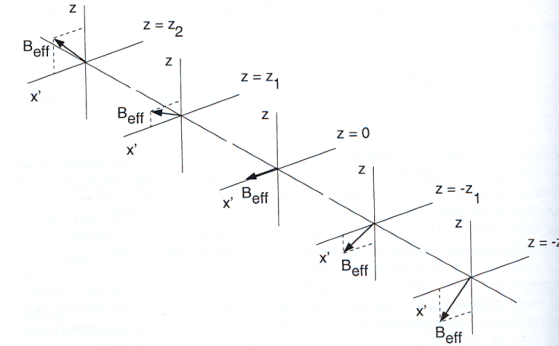
Let $\mathbf{B}_{rot} = B_1(t)\mathbf{i} + (B_0 + \gamma G_z z)\mathbf{k}$

$$\begin{aligned} \mathbf{B}_{eff} &= \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma} \\ &= B_1(t)\mathbf{i} + \left(B_0 + \gamma G_z z - \frac{\omega}{\gamma} \right) \mathbf{k} \end{aligned}$$

If $\omega = \omega_0$

$$\mathbf{B}_{eff} = B_1(t)\mathbf{i} + (\gamma G_z z)\mathbf{k}$$

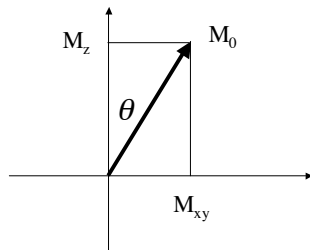
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Nishimura 1996

Small Tip Angle Approximation



For small θ

$$M_z = M_0 \cos \theta \approx M_0$$

$$M_{xy} = M_0 \sin \theta \approx M_0 \theta$$

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Recall that in the rotating frame, flip angle $\theta = \gamma \int_0^t B_1(s) ds$

Define $\omega(z)$ as the Larmor Frequency at each location z referenced to ω_0
The effective field felt in each spin's rotating frame of reference is:

$$B_1^e(t) = B_1(t) \exp(j\omega(z)t)$$

Therefore the flip angle in each spin's frame of reference is

$$\theta(t, z) = \gamma \int_0^t \exp(j\omega(z)s) B_1(s) ds$$

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With respect to the on - resonance frame of reference, there is also a relative phase shift of $\exp(-j\omega(z)t)$, so that

$$\theta(t,z) = \gamma \exp(-j\omega(z)t) \int_0^t \exp(j\omega(z)s) B_1(s) ds$$

Applying small angle approximation leads to

$$M_r(t,z) \approx jM_0 \theta(t,z) = jM_0 \gamma \exp(-j\omega(z)t) \int_0^t \exp(j\omega(z)s) B_1(s) ds$$

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Small Tip Angle Approximation

$$M_r(t,z) = jM_0 \exp(-j\omega(z)t) \int_0^t \exp(j\omega(z)s) \omega_1(s) ds$$

For symmetric pulse of length τ

$$\begin{aligned} M_r(\tau,z) &= jM_0 \exp(-j\omega(z)\tau/2) \int_{-\tau/2}^{\tau/2} \exp(j2\pi f(z)s) \omega_1(s + \tau/2) ds \\ &= jM_0 \exp(-j\omega(z)\tau/2) F\{\omega_1(t + \tau/2)\} \Big|_{f=-f(z)=-\frac{\gamma}{2\pi}G_z z} \end{aligned}$$

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Small Tip Angle Example

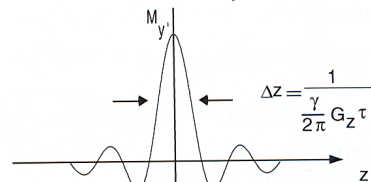
$$B_1(t) = B_{\text{rect}}\left(\frac{t-\tau/2}{\tau}\right)$$

$$M_r(\tau,z) = jM_0 \exp(-j\omega(z)\tau) \int_0^\tau \exp(j\omega(z)s) \omega_1 \text{rect}\left(\frac{s-\tau/2}{\tau}\right) ds$$

$$= jM_0 \exp(-j\omega(z)\tau/2) F_{1D}\left(\omega_1 \text{rect}\left(\frac{t}{\tau}\right)\right) \Big|_{f=-\frac{\gamma}{2\pi}G_z z}$$

$$= jM_0 \exp(-j\omega(z)\tau/2) \omega_1 \tau \text{sinc}(f\tau)$$

$$= jM_0 \exp(-j\omega(z)\tau/2) \omega_1 \tau \text{sinc}\left(\frac{\gamma G_z \tau}{2\pi} z\right)$$



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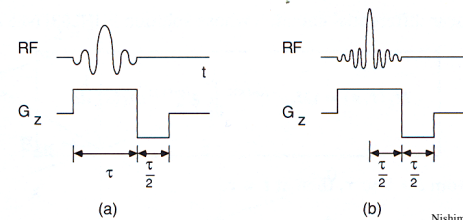
Nishimura 1996

Refocusing

$$M_r(3\tau/2,z) = \exp(j\omega(z)\tau/2) M_r(\tau,z)$$

$$= jM_0 \exp(j\omega(z)\tau/2) \exp(-j\omega(z)\tau/2) F\{\omega_1(t + \tau/2)\} \Big|_{f=-\frac{\gamma}{2\pi}G_z z}$$

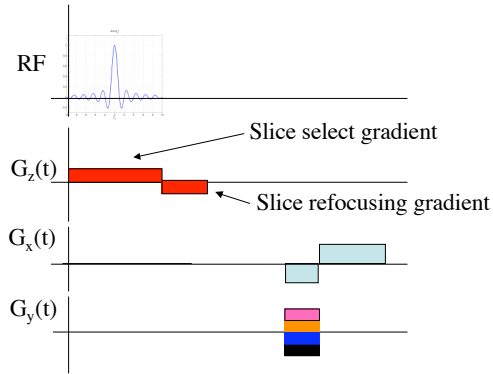
$$= jM_0 F\{\omega_1(t + \tau/2)\} \Big|_{f=-\frac{\gamma}{2\pi}G_z z}$$



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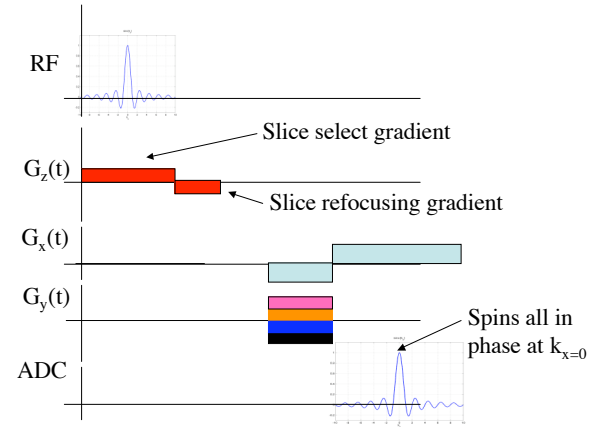
Nishimura 1996

Slice Selection



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Gradient Echo



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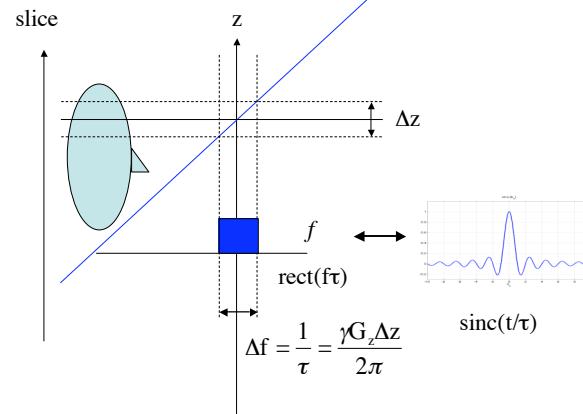
Small Tip Angle Example

$$\begin{aligned}
 B_1(t + \tau/2) &= A \operatorname{sinc}(t/\tau) \left(0.5 + 0.46 \cos\left(\frac{2\pi t}{\tau}\right) \right) \\
 &= A \operatorname{sinc}(t/\tau) w(t) \\
 F(B_1(t + \tau/2)) &= A\tau \operatorname{rect}(f\tau) * W(f) \\
 &= A\tau \operatorname{rect}\left(\frac{\gamma G_z z \tau}{2\pi}\right) * W\left(-\frac{\gamma G_z z}{2\pi}\right)
 \end{aligned}$$

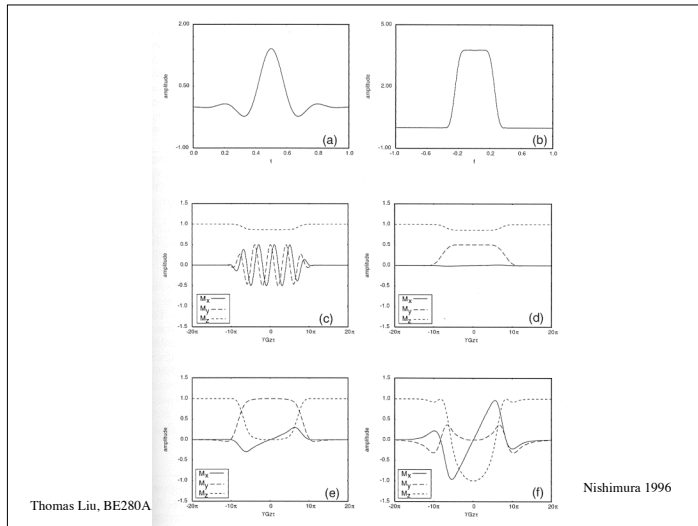
$$\text{Width of the rect function is } \Delta z = \frac{2\pi}{\gamma G_z \tau}$$

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Slice Selection



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Example

$\Delta z = 5 \text{ mm}; \tau = 400 \text{ } \mu\text{sec}; \theta = \pi/2$

$$G_z = \frac{2\pi}{\gamma \Delta z \tau} = \frac{1}{(4257 \text{ Hz/G})(0.5 \text{ cm})(400 \times 10^{-6})} = 1.175 \text{ G/cm}$$

$$\theta \approx \gamma \int_0^T B_1 \text{sinc}\left(\frac{s-T/2}{\tau}\right) ds \approx \gamma B_1 \cdot (\text{area of sinc}) = \gamma B_1 \tau$$

$$B_1 = \frac{\theta}{\gamma \tau} = \frac{\pi/2}{2\pi(4257 \text{ Hz/G})(400 \times 10^{-6})} = 0.1468 \text{ G}$$

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Cardiac Tagging

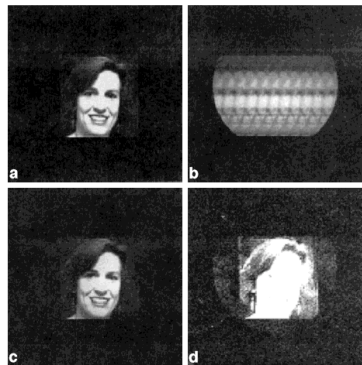
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Excitation k-space

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Pauly et al 1989

Excitation k-space



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Panych MRM 1999