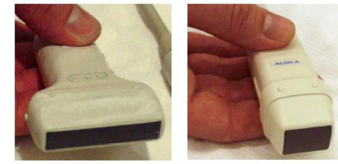
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2008
Ultrasound Lecture 1

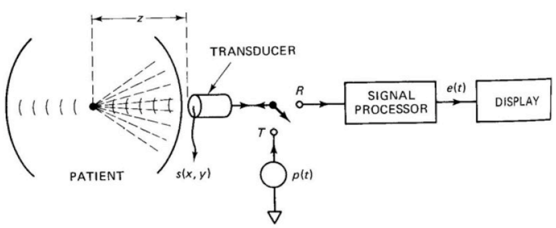
Acuson Sequoia



Sonosite 180

From Suetens 2002

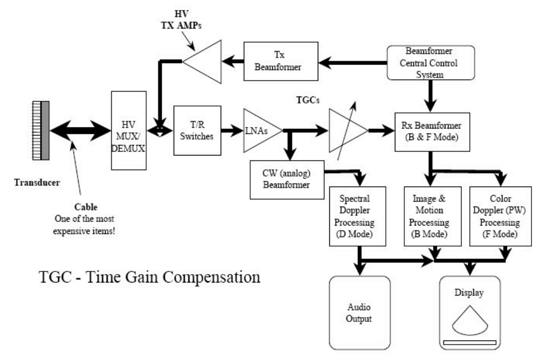
Basic System



Echo occurs at $t=2z/c$ where c is approximately 1500 m/s or 1.5 mm/μs

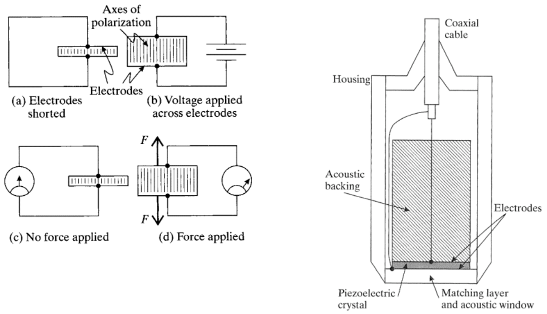
Macovski 1983

Basic System



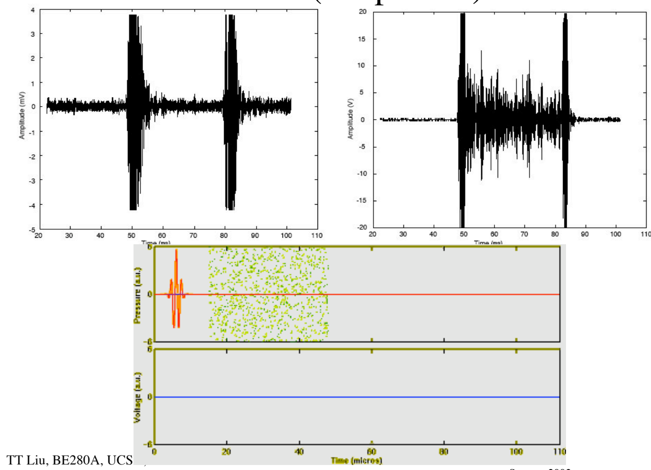
Brunner 2002

Transducer



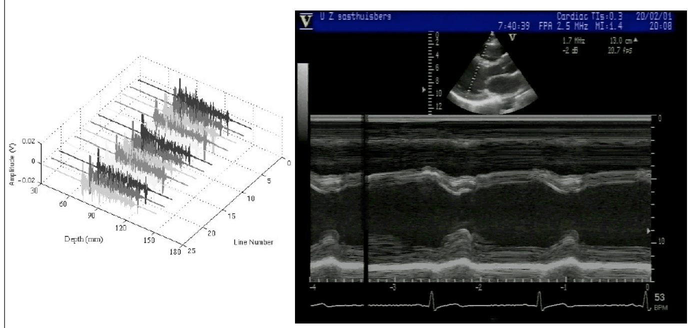
Prince and Links 2006

A-Mode (Amplitude)



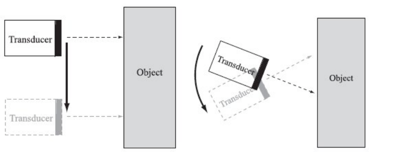
Seutens 2002

M-Mode (Motion)

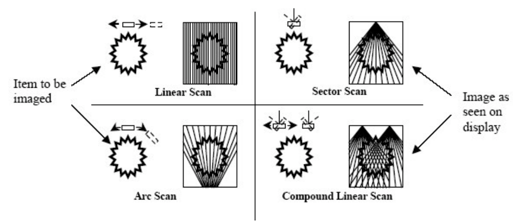


Seutens 2002

B-Mode (Brightness)

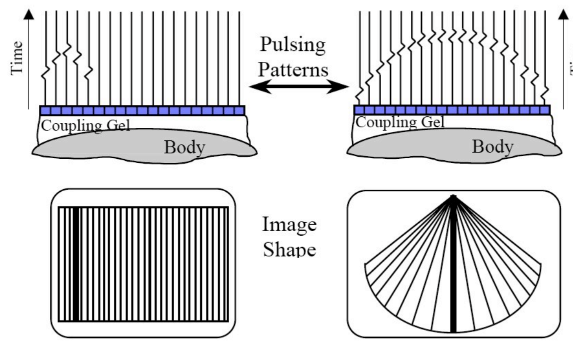


Seutens 2002



Brunner 2002

B-Mode (Brightness)



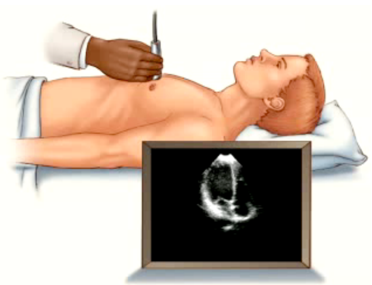
Brunner 2002

B-Mode

2002

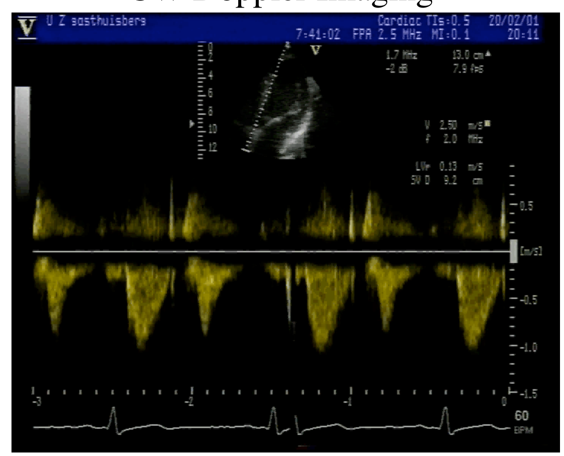
Mayo Clinic

B-Mode



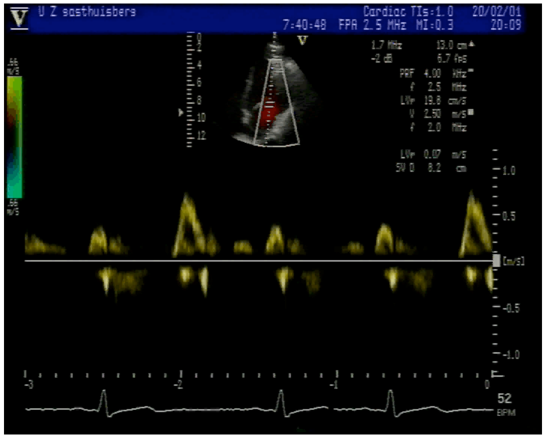
Credit: Mayo Clinic

CW Doppler Imaging



Seutens 2002

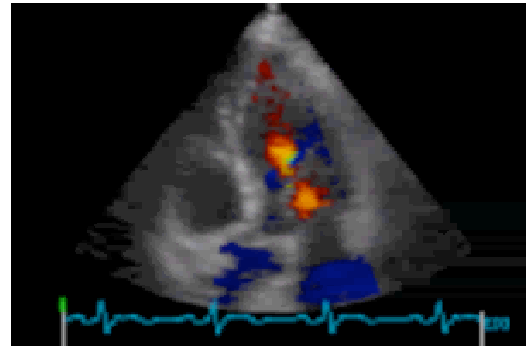
PW Doppler Imaging



TT Liu, BE280A, UCSD, Fall 2008

Seutens 2002

Color Doppler Imaging



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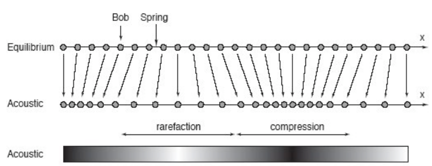
Acoustic Waves



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Speed of Sound

$$c = \sqrt{\frac{1}{\kappa\rho}} \quad [\mathrm{m\ s^{-1}}]$$

$\kappa =$ compressibility $[\mathrm{m\ s^{2}\ kg^{-1}}] = [1/\mathrm{Pascal}]$

$\rho =$ density $[\mathrm{kg\ m^{-3}}]$

Material	Density	Speed m/s
Air	1.2	330
Water	1000	1480
Bone	1380-1810	4080
Fat	920	1450
Liver	1060	1570

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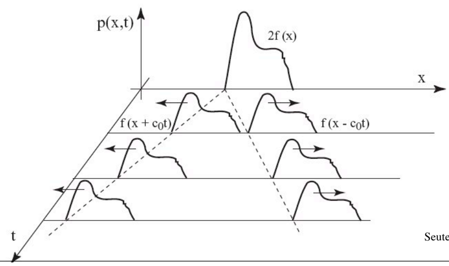
Acoustic Wave Equation

$$\nabla^2 p = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

Solutions are of the form

$$p(x,t) = A_1 f_1(x - ct) + A_2 f_2(x + ct)$$



Seutens 2002

Plane Waves

$$p(z,t) = \cos(k(z-ct)) = \cos\left(\frac{2\pi}{\lambda}(z-ct)\right) = \cos\left(\frac{2\pi f}{c}(z-ct)\right) = \cos(2\pi f(z/c - t))$$

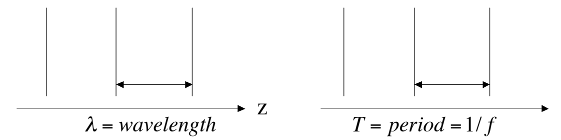
$$p(z,t) = \exp(jk(z-ct))$$

$k = wavenumber = \frac{2\pi}{\lambda} = 2\pi k_z$

$\lambda = wavelength = \frac{c}{f}$

$f = frequency$ [cycles/sec]

$T = period = \frac{1}{f}$



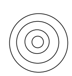
Spherical Waves

Outward wave → Inward wave

$$p(r,t) = \frac{1}{r}\phi(t - r/c) + \frac{1}{r}\phi(t + r/c)$$

Outward wave

$$p(r,t) = \frac{1}{r}\exp(j2\pi f(t - r/c))$$



Impedance

$$\text{Impedance } Z = \frac{\text{Pressure}}{\text{Velocity}} = \frac{P}{v} = \rho c = \sqrt{\frac{\rho}{\kappa}}$$

density kg/m^3

speed of sound
Brain 1541 m/s
Liver 1549
Skull bone 4080 m/s
Water 1480 m/s

Note: particle velocity and speed of sound are not the same!

Impedance

$$Z = \rho c = \sqrt{\frac{\rho}{\kappa}}$$

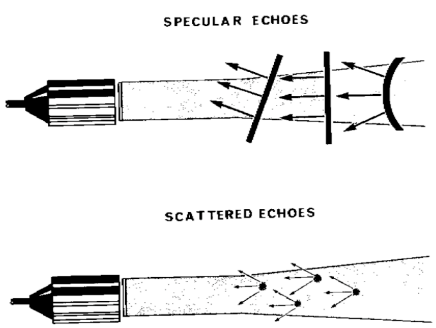
Material	Density	Speed m/s	Z (kg/m^2/s)
Air	1.2	330	0.0004
Water	1000	1480	1.5
Bone	1380-1810	4080	3.75-7.38
Fat	920	1450	1.35
Liver	1060	1570	1.64-1.68

Acoustic Intensity

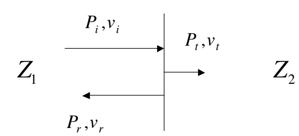
$$I = pv$$
$$= \frac{p^2}{Z}$$

Also called acoustic energy flux.
Analogous to electric power

Echos



Specular Reflection



Material	Reflectivity
Brain-skull	0.66
Fat-muscle	0.10
Muscle-blood	0.03
Soft-tissue-air	.9995

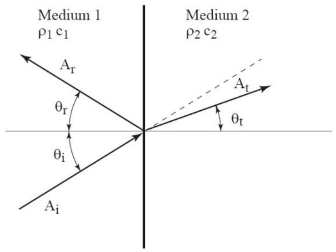
$v_i - v_r = v_t$ (velocity boundary condition)

$$\frac{P_i}{Z_1} - \frac{P_r}{Z_1} = \frac{P_t}{Z_2}$$

$P_i + P_r = P_t$ (pressure boundary condition)

$$R = \frac{P_r}{P_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \approx \frac{\Delta Z}{Z_0}$$

Reflection and Refraction



Snell's Law

$$\frac{\sin\theta_i}{c_1} = \frac{\sin\theta_r}{c_1} = \frac{\sin\theta_t}{c_2}$$

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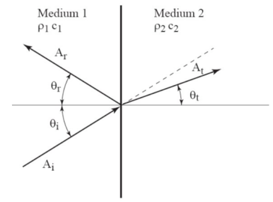
Reflection and Refraction



$$v_i\cos\theta_i = v_r\cos\theta_r + v_t\cos\theta_t$$

$$\frac{p_i}{Z_1}\cos\theta_i = \frac{p_r}{Z_1}\cos\theta_r + \frac{p_t}{Z_2}\cos\theta_t$$

$$p_i + p_r = p_t$$

Pressure Reflectivity $$R = \frac{p_r}{p_i} = \frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_2\cos\theta_i + Z_1\cos\theta_t}$$

Pressure Transmittivity $$T = \frac{p_t}{p_i} = \frac{2Z_2\cos\theta_i}{Z_2\cos\theta_i + Z_1\cos\theta_t}$$

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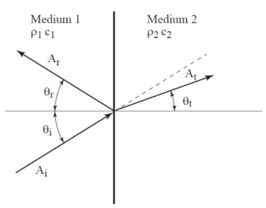
Reflection and Refraction



Intensity Reflectivity $$R_I = \frac{I_r}{I_i} = \frac{p_r^2}{p_i^2} = \left(\frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_2\cos\theta_i + Z_1\cos\theta_t}\right)^2$$

Intensity Transmittivity $$T_I = \frac{I_t}{I_i} = \frac{p_t^2 Z_1}{p_i^2 Z_2} = \frac{4Z_1Z_2\cos^2\theta_i}{(Z_2\cos\theta_i + Z_1\cos\theta_t)^2}$$

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Example

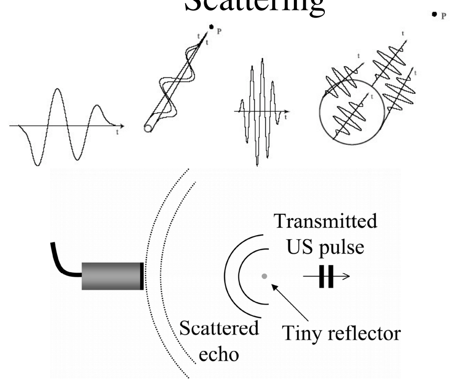
Example: Fat/liver interface at normal incidence

$Z_{fat} = 1.35\times10^{-6}$ kg m^{-2} s^{-1}

$Z_{liver} = 1.66\times10^{-6}$kg m^{-2} s^{-1}

$$R_I = \left(\frac{Z_{liver} - Z_{fat}}{Z_{liver} + Z_{fat}}\right)^2 = 0.103$$

Scattering



Point scatterers retransmit the incident wave equally in all direction (e.g. isotropic scattering).

Attenuation

Loss of acoustic energy during propagation.

Conversion of acoustic energy into heat.

$$p(z,t) = A_z f(t - c/z)$$
$$= A_0 \exp(-\mu_a z) f(t - c/z)$$

Amplitude attenuation factor

$$\mu_a = -\frac{1}{z}\ln\frac{A_z}{A_0} \quad : \text{ units } = \text{ nepers/cm}$$

$$\alpha = -20\frac{1}{z}\log_{10}\frac{A_z}{A_0} = 20\mu_a \log_{10}(e) \approx 8.7\mu_a \quad : \text{ dB/cm}$$

Attenuation coefficient

Attenuation

$\alpha(f) = \alpha_0 f^n$

For frequencies used in medical ultrasound, $n \approx 1$.

$\alpha(f) \approx \alpha_0 f$

Material	α_0 [dB/cm/MHz]
fat	0.63
liver	0.94
Cardiac muscle	1.8
bone	20.0

Example

Example : Fat at 5 MHz

Attenuation coefficient $= 5\text{MHz} \times 0.63$ dB/cm/MHz

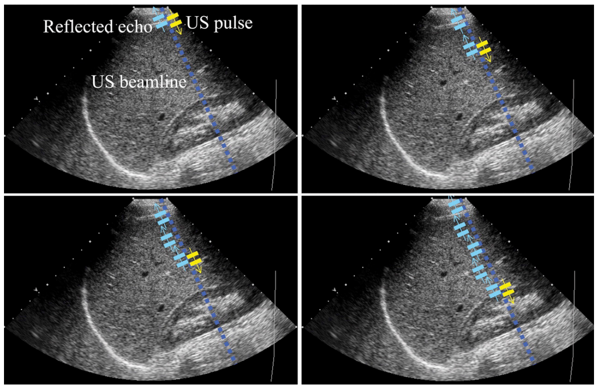
$= 3.15$dB/cm

After 4 cm, attenuation $= 4 * 3.15 = 12.6$ dB

Relative amplitude is $10^{(-12.6/20)} = 0.2344$

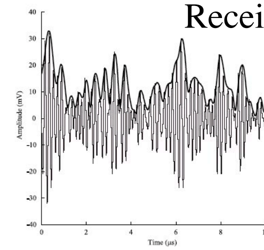
Recall dB $\equiv 20\log_{10}(A_z / A_0)$

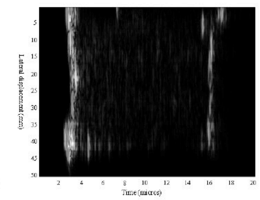
Received signal



http://radiographics.rsnajnls.org/content/vol23/issue4/images/large/g03jl25c1x.jpeg

Received signal





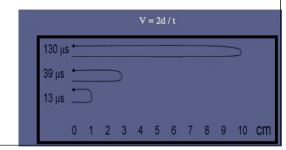
$$e(t) = K\int\int\int \frac{e^{-2\alpha z}}{z} R(x,y,z)s(x,y)p(t-2z/c)\,dxdydz$$

Attenuation

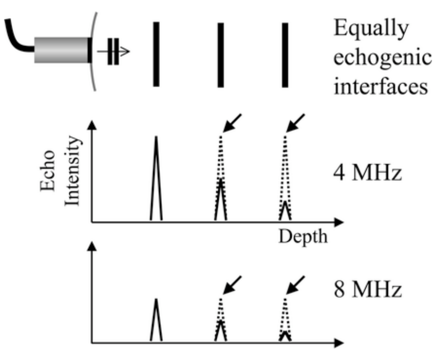
Reflection/Scattering

Beam width

Pulse



Attenuation Correction



Attenuation Correction and Signal Equation

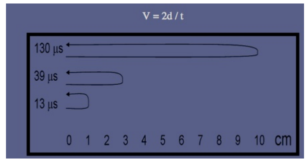
$$\begin{aligned} e(t) &= K\int\int\int \frac{e^{-2\alpha z}}{z} R(x,y,z)s(x,y)p(t-2z/c)\,dxdydz \\ &\approx K\frac{e^{-\alpha ct}}{ct/2}\int\int\int R(x,y,z)s(x,y)p(t-2z/c)\,dxdydz \end{aligned}$$

$$\begin{aligned} e_c(t) &= cte^{\alpha ct}e(t) \\ &\approx K\int\int\int R(x,y,z)s(x,y)p(t-2z/c)\,dxdydz \\ &= \frac{c}{2}\int\int\int R(x,y,c\tau/2)s(x,y)p(t-\tau)\,dxdyd\tau \end{aligned}$$

Depth Response

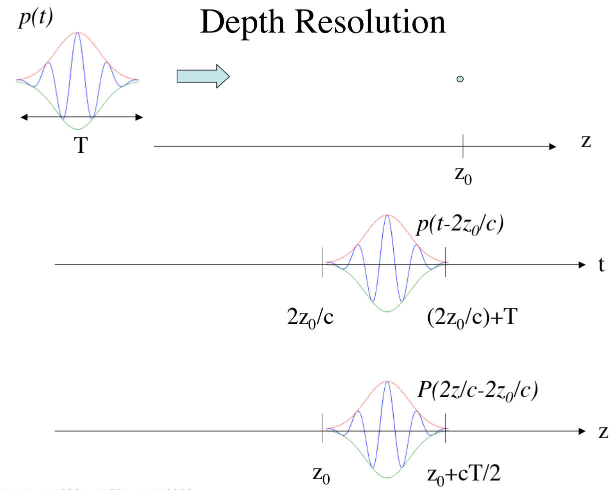
Depth response

$$p(t - 2z_0/c) = p(2z/c - 2z_0/c)$$
$$= p\left(\frac{2(z - z_0)}{c}\right)$$

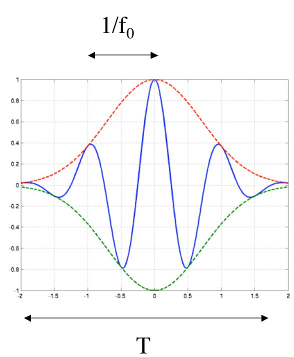


Therefore impulse response is simply
$p(t)$ in the time domain or
$p(2z/c)$ in the spatial domain

Depth Resolution



Depth Resolution



$p(t) = p(2z/c)$ determines the depth resolution

Pulses are of the form $a(t)\cos(2\pi f_0 t + \theta)$ where $a(t)$ is the envelope function and f_0 is the resonant frequency of the transducer.

The duration of T of $a(t)$ is typically chosen to be about 2 or 3 periods (e.g. $T = 3/f_0$).
If the duration is too short, the bandwidth of the pulse will be very large and much of its power will be attenuated.

The depth resolution is approximately
$\Delta z = cT/2 \approx 1.5c/f_0 = 1.5\lambda$.

Depth Resolution

The depth resolution is approximately $\Delta z = cT/2 \approx 1.5c/f_0 = 1.5\lambda$.

Example:
For $f_0 = 5$ MHz, $\lambda = c/f = (1500m/s)/(5\times10^6 Hz) = 0.3mm$
$\Delta z = 1.5\lambda = 0.45$ mm

Trade-off
Higher $f_0 \Rightarrow$ Smaller $\Delta z \Rightarrow$ but more attenuation

Example: Assume 1dB/cm/MHz
For 10 cm depth, 20 cm roundtrip path length.
At 1 MHz 20 dB of attenuation $\Rightarrow$ Attenuation = 0.1
At 10 MHz 200 dB of attenuation $\Rightarrow$ Attenuation = $1\text{x}10^{-10}$

Depth of Penetration

Assume system can handle L dB of loss, then

$$L = 20\log_{10}\left(\frac{A_z}{A_0}\right)$$

We also have the definition

$$\alpha = -\frac{1}{z}20\log_{10}\left(\frac{A_z}{A_0}\right)$$

and the approximation

$$\alpha = \alpha_0 f$$

Total range a wave can travel before attenuation L is

$$z = \frac{L}{\alpha_0 f}$$

Depth of penetration is

$$d_p = \frac{L}{2\alpha_0 f}$$

Depth of Penetration

Assume L = 80 dB; α_0= 1dB/cm/MHz

Frequency (MHz)	Depth of Penetration(cm)
1	40
2	20
3	13
5	8
10	4
20	2

Pulse Repetition and Frame Rate

Need to wait for echoes to return before transmitting new pulse

Pulse repetition interval is

$$T_R \geq \frac{2d_p}{c}$$

Pulse repetition rate is

$$f_R = \frac{1}{T_R}$$

If N pulses are required to form an image, then the frame rate is

$$F = \frac{1}{NT_R}$$

Example

$N = 256,\ L = 80dB,\ c = 1540m/s,\ \alpha_0 = 1dB/cm/MHz$

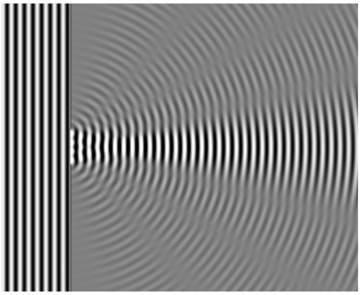
What frequency should be used to achieve a frame rate of 15 frame/sec?

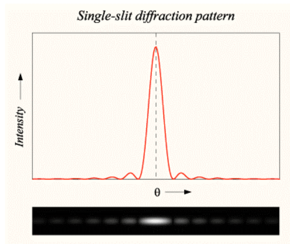
$$T_R = \frac{1}{FN} = 0.26ms$$

$$T_R \geq \frac{2d_p}{c} = \frac{L}{\alpha_0 fc}$$

$$f \geq \frac{L}{acT_R} = 1.99MHz$$

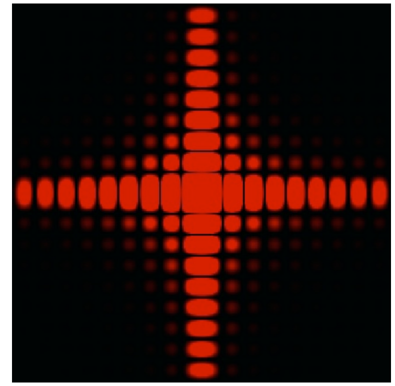
Single-slit Diffraction





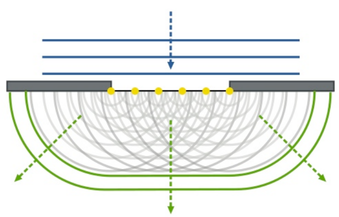
Source:wikipedia

Square Aperture Diffraction



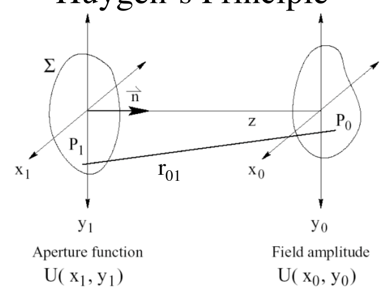
Source:wikipedia

Huygen's Principle



Source:wikipedia

Huygen's Principle



Wavenumber

$$k = \frac{2\pi}{\lambda}$$

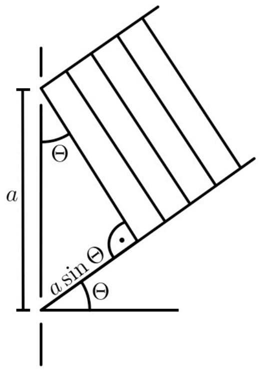
$$U(P_0) = \iint_{\Sigma} h(P_0, P_1) U(P_1) ds$$

$$\text{where } h(P_0, P_1) = \frac{1}{j\lambda} \frac{\exp(jkr_{01})}{r_{01}} \cos(\vec{n}, \vec{r}_{01})$$

Obliquity Factor

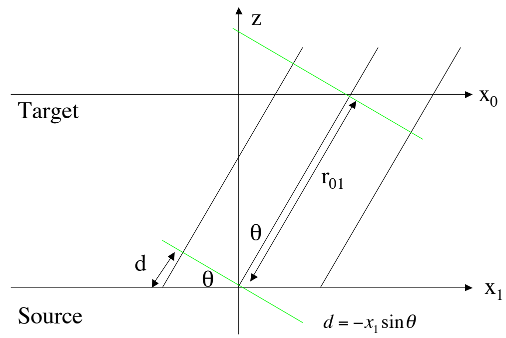
Anderson and Trahey 2000

Two-Slit Interference



Source:wikipedia

Plane Wave (Fraunhofer) Approximation



$$d = -x_1 \sin\theta$$

$$\sin\theta = \frac{x_0}{r_{01}} \approx \frac{x_0}{z}$$

$$d \approx -\frac{x_0 x_1}{z}$$

Assume overall distance for phase calculation is approximately z+d

Plane Wave Approximation

$$\frac{1}{r}\exp(jkr) \approx \frac{1}{z}\exp\big(jk(z+d)\big) = \frac{1}{z}\exp\left(j\frac{2\pi}{\lambda}\left(z - \frac{x_0 x_1}{z}\right)\right)$$

$$\begin{aligned}
U(x_0) &= \int_{-\infty}^{\infty} s(x_1)\frac{1}{r}\exp(jkr)dx_1 \\
&\approx \int_{-\infty}^{\infty} s(x_1)\frac{1}{z}\exp\left(j\frac{2\pi}{\lambda}\left(z - \frac{x_0 x_1}{z}\right)\right)dx_1 \\
&= \frac{1}{z}\exp\left(j\frac{2\pi z}{\lambda}\right)\int_{-\infty}^{\infty} s(x_1)\exp\left(-\frac{j2\pi x_0 x_1}{\lambda z}\right)dx_1 \\
&= \frac{1}{z}\exp\left(j\frac{2\pi z}{\lambda}\right)\int_{-\infty}^{\infty} s(x_1)\exp\left(-j2\pi k_x x_1\right)dx_1 \\
&= \frac{1}{z}\exp\left(j\frac{2\pi z}{\lambda}\right)F\left[s(x)\right]\Big|_{k_x = \frac{x_0}{\lambda z}}
\end{aligned}$$

Plane Wave Approximation

In general

$$U(x_0, y_0) = \frac{1}{z}\exp\left(j\frac{2\pi z}{\lambda}\right)F\left[s(x,y)\right]\Big|_{k_x = \frac{x_0}{\lambda z}, k_y = \frac{y_0}{\lambda z}}$$

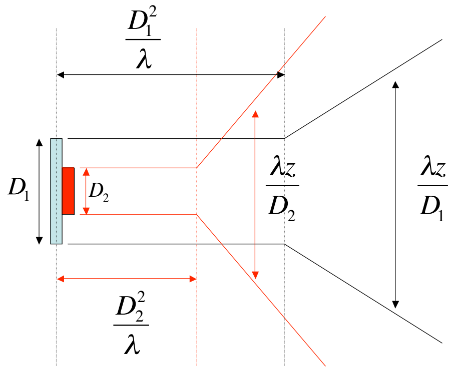
Example

$$s(x,y) = rect(x/D)rect(y/D)$$

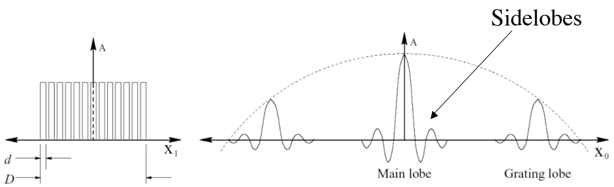
$$\begin{aligned}
U(x_0, y_0) &= \frac{1}{z}\exp(jkz)D^2 \operatorname{sinc}\left(Dk_x\right)\operatorname{sinc}\left(Dk_y\right) \\
&= \frac{1}{z}\exp(jkz)D^2 \operatorname{sinc}\left(D\frac{x_0}{\lambda z}\right)\operatorname{sinc}\left(Dk_y\frac{y_0}{\lambda z}\right)
\end{aligned}$$

Zeros occur at $x_0 = \frac{n\lambda z}{D}$ and $y_0 = \frac{n\lambda z}{D}$

Beamwidth of the sinc function is $\frac{\lambda z}{D}$



Example



$$rect\left(\frac{x}{D}\right)\left[rect\left(\frac{x}{d}\right)*\frac{1}{d}comb\left(\frac{x}{d}\right)\right]\Leftrightarrow D\operatorname{sinc}(Dk_x)*\left[d\operatorname{sinc}(dk_x)comb(dk_x)\right]$$

Question: What should we do to reduce the sidelobes?

Anderson and Trahey 2000