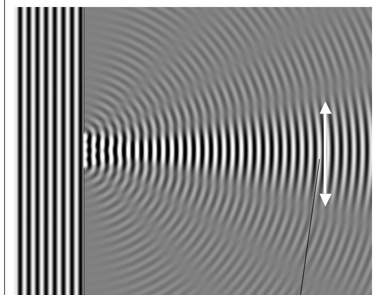
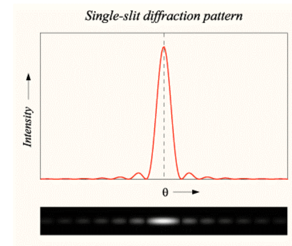
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2008
Ultrasound Lecture 2

Single-slit Diffraction





Beamwidth $\frac{\lambda z}{D}$

Source:wikipedia

Plane Wave Approximation

In general

$$U(x_0,y_0)=\frac{1}{z}\exp\left(j\frac{2\pi z}{\lambda}\right)F\left[s(x,y)\right]\Big|_{k_x=\frac{x_0}{\lambda z},k_y=\frac{y_0}{\lambda z}}$$

Example

$$s(x,y)=rect(x/D)rect(y/D)$$

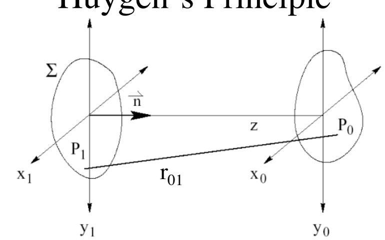
$$U(x_0,y_0)=\frac{1}{z}\exp(jkz)D^2\,\mathrm{sinc}\left(Dk_x\right)\mathrm{sinc}\left(Dk_y\right)$$

$$=\frac{1}{z}\exp(jkz)D^2\,\mathrm{sinc}\left(D\frac{x_0}{\lambda z}\right)\mathrm{sinc}\left(Dk_y\frac{y_0}{\lambda z}\right)$$

Zeros occur at $x_0=\frac{n\lambda z}{D}$ and $y_0=\frac{n\lambda z}{D}$

Beamwidth of the sinc function is $\frac{\lambda z}{D}$

Huygen's Principle



Aperture function
$U(x_1, y_1)$

Field amplitude
$U(x_0, y_0)$

Wavenumber

$$k=\frac{2\pi}{\lambda}$$

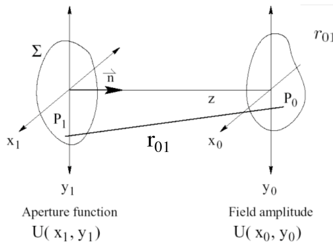
$$U(P_0)=\iint_{\Sigma} h(P_0,P_1)U(P_1)ds$$

$$\text{where } h(P_0,P_1)=\frac{1}{j\lambda}\frac{\exp(jkr_{01})}{r_{01}}\cos(\vec{n},\vec{r}_{01})$$

Oliquity Factor

Anderson and Trahey 2000

Small-Angle (paraxial) Approximation



$$r_{01} = \sqrt{z^2 + (x_1 - x_0)^2 + (y_1 - y_0)^2}$$

$$cos(\overrightarrow{n}, \overrightarrow{r}_{01}) \approx 1$$

$$r_{01} \approx z$$

$$h(x_0, y_0; x_1, y_1) \approx \frac{1}{j\lambda z} \exp(jkr_{01})$$

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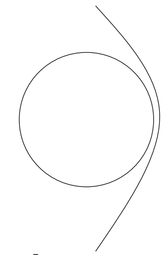
Anderson and Trahey 2000

Fresnel Approximation

$$\begin{aligned} r_{01} &= \sqrt{z^2 + (x_1 - x_0)^2 + (y_1 - y_0)^2} \\ &= z\sqrt{1 + \left(\frac{(x_1 - x_0)}{z}\right)^2 + \left(\frac{(y_1 - y_0)^2}{z}\right)} \end{aligned}$$

$$\approx z\left[1 + \frac{1}{2}\left(\frac{(x_1 - x_0)}{z}\right)^2 + \frac{1}{2}\left(\frac{(y_1 - y_0)}{z}\right)^2\right]$$



Approximates spherical wavefront with a parabolic phase profile

$$h(x_0, y_0; x_1, y_1) \approx \frac{\exp(jkz)}{j\lambda z} \exp\left[\frac{jk}{2z}\left[(x_1 - x_0)^2 + (y_1 - y_0)^2\right]\right]$$

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Fresnel Approximation

$$\begin{aligned} U(x_0, y_0) &= \iint \frac{1}{j\lambda r_{01}} \exp(jkr_{01}) s(x_1, y_1) dx_1 dy_1 \\ &\approx \iint \frac{1}{j\lambda z} \exp\left(jk\left(z^2 + (x_1 - x_0)^2 + (y_1 - y_0)^2\right)\right) s(x_1, y_1) dx_1 dy_1 \\ &\approx \iint \frac{\exp(jkz)}{j\lambda z} \exp\left(\frac{jk}{2z}\left((x_1 - x_0)^2 + (y_1 - y_0)^2\right)\right) s(x_1, y_1) dx_1 dy_1 \\ &= \frac{\exp(jkz)}{j\lambda z}\left(s(x_0, y_0) ** \exp\left(\frac{jk}{2z}\left(x_0^2 + y_0^2\right)\right)\right) \end{aligned}$$

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Fresnel Zone

$$\begin{aligned} U(x_0, y_0) &\approx \iint \frac{\exp(jkz)}{j\lambda z} \exp\left(\frac{jk}{2z}\left((x_1 - x_0)^2 + (y_1 - y_0)^2\right)\right) s(x_1, y_1) dx_1 dy_1 \\ &= \iint \frac{\exp(jkz)}{j\lambda z} \exp\left(\frac{jk}{2z}\left((x_1^2 + y_1^2) + (x_0^2 + y_0^2) - 2(x_1 x_0 + y_1 y_0)\right)\right) s(x_1, y_1) dx_1 dy_1 \end{aligned}$$

$$= \frac{\exp(jkz)}{j\lambda z} \exp\left(\frac{jk}{2z}\left(x_0^2 + y_0^2\right)\right) \times$$

$$\iint \frac{\exp(jkz)}{j\lambda z} \exp\left(\frac{jk}{2z}\left(x_1^2 + y_1^2\right)\right) s(x_1, y_1) \exp\left(-\frac{jk}{z}\left(x_1 x_0 + y_1 y_0\right)\right) dx_1 dy_1$$

$$= \frac{\exp(jkz)}{j\lambda z} \exp\left(\frac{jk}{2z}\left(x_0^2 + y_0^2\right)\right) F\left[\exp\left(\frac{jk}{2z}\left(x_1^2 + y_1^2\right)\right) s(x_1, y_1)\right]$$

Phase across transducer face

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Fraunhofer Condition

Phase term due to position on transducer is $\frac{k}{2z}\left(x_1^2 + y_1^2\right)$

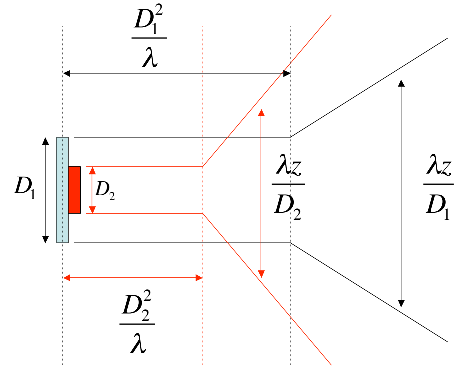
Far-field condition is

$$\frac{k}{2z}\left(x_1^2 + y_1^2\right) << 1$$

$$z >> \frac{k}{2}\left(x_1^2 + y_1^2\right) = \frac{\pi}{\lambda}\left(x_1^2 + y_1^2\right)$$

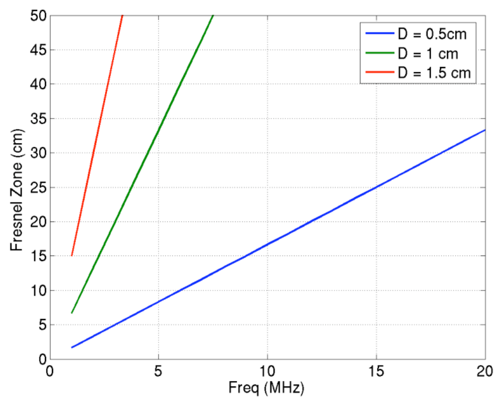
For a square DxD transducer, $x_1^2 + y_1^2 = D^2/2$

$$z >> \frac{\pi D^2}{2\lambda} \approx \frac{D^2}{\lambda}$$



$$\left.\frac{\lambda z}{D}\right|_{z=D^2/\lambda} = D$$

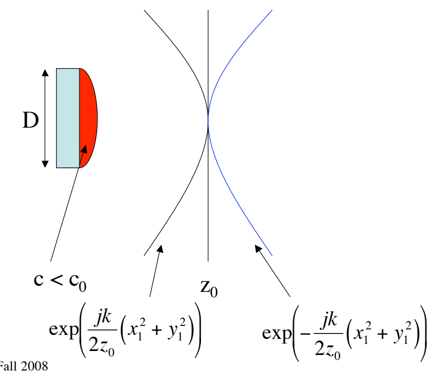
Fresnel Zone



Anderson and Trahey 2000

Focusing in Fresnel Zone

$$U(x_0,y_0) = \frac{\exp(jkz)}{j\lambda z}\exp\left(\frac{jk}{2z}\left(x_0^2 + y_0^2\right)\right)F\left[\exp\left(\frac{jk}{2z}\left(x_1^2 + y_1^2\right)\right)s(x_1,y_1)\right]$$



Focusing in Fresnel Zone

$$U(x_0,y_0)=\frac{\exp(jkz)}{j\lambda z}\exp\left(\frac{jk}{2z}\left(x_0^2+y_0^2\right)\right)F\left[\exp\left(\frac{jk}{2z}\left(x_1^2+y_1^2\right)\right)s(x_1,y_1)\right]$$

Make

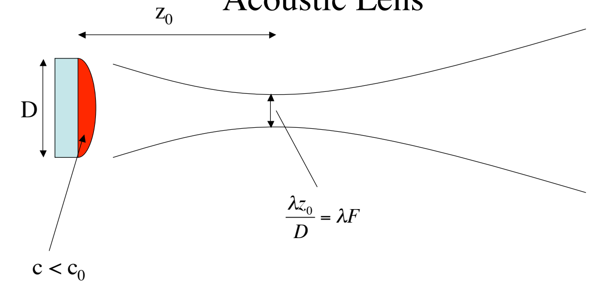
$$s(x_1,y_1)=s_0(x_1,y_1)\exp\left(-\frac{jk}{2z_0}\left(x_1^2+y_1^2\right)\right)$$

At the focal depth $z = z_0$

$$U(x_0,y_0)=\frac{\exp(jkz_0)}{j\lambda z_0}\exp\left(\frac{jk}{2z_0}\left(x_0^2+y_0^2\right)\right)F\left[s(x_1,y_1)\right]$$

Beamwidth at the focal depth is: $\frac{\lambda z_0}{D}$

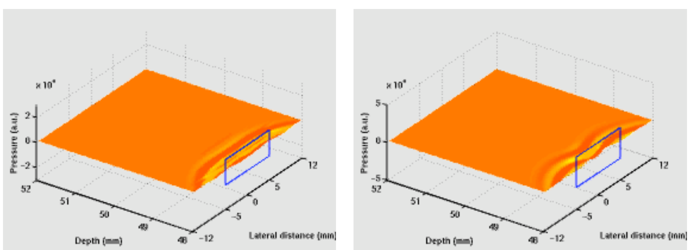
Acoustic Lens

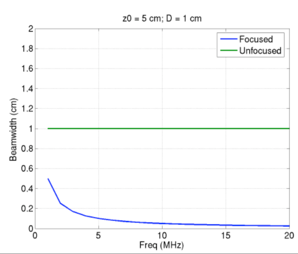


At the focal depth $z = z_0$

$$U(x_0,y_0)=\frac{\exp(jkz_0)}{j\lambda z_0}\exp\left(\frac{jk}{2z_0}\left(x_0^2+y_0^2\right)\right)F\left[\exp\left(\frac{jk}{2z_0}\left(x_1^2+y_1^2\right)\right)\exp\left(-\frac{jk}{2z_0}\left(x_1^2+y_1^2\right)\right)s(x_1,y_1)\right]$$

$$=\frac{\exp(jkz_0)}{j\lambda z_0}\exp\left(\frac{jk}{2z_0}\left(x_0^2+y_0^2\right)\right)F\left[s(x_1,y_1)\right]$$





Depth of Focus

When $z \neq z_0$, the phase term is $\Delta\Phi=\exp\left(-\frac{jk}{2z_0}\left(x_1^2+y_1^2\right)\right)\exp\left(-\frac{jk}{2z}\left(x_1^2+y_1^2\right)\right)$

and the lens is not perfectly focused.

Consider variation in the x - direction.

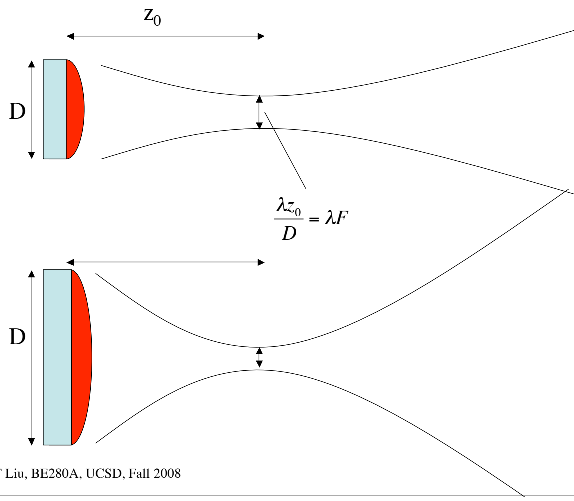
$$\Delta\Phi=\frac{kx^2}{2}\left(\frac{1}{z}-\frac{1}{z_0}\right)$$

For transducer of size D, $\frac{x^2}{2}=\frac{D^2}{8}$

If we want $\left|\Delta\Phi\right|=\left|\frac{\pi D^2}{4\lambda}\left(\frac{1}{z}-\frac{1}{z_0}\right)\right|<1$ radian then

$$\left|\frac{1}{z}-\frac{1}{z_0}\right|<\frac{4\lambda}{\pi D^2}\approx\frac{\lambda}{D^2}\Rightarrow\quad\frac{\Delta z}{z_0^2}<\frac{\lambda}{D^2}$$

The larger the D, the smaller the depth of focus.



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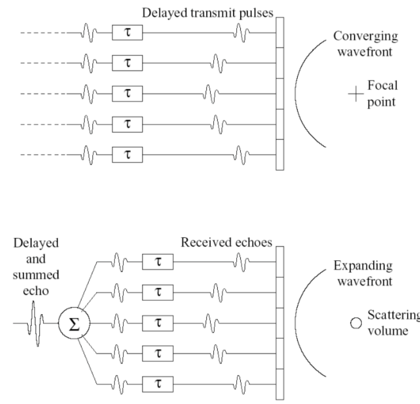
Focusing with Phased Array



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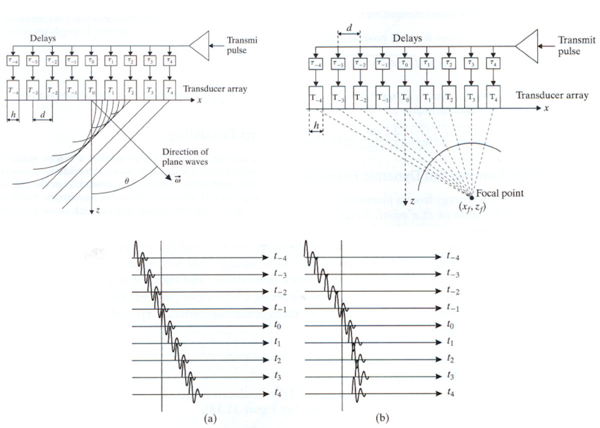
Anderson and Trahey 2000

Focusing and Steering



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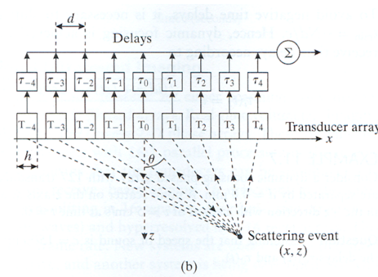
Prince and Links 2005

Dynamic Focusing



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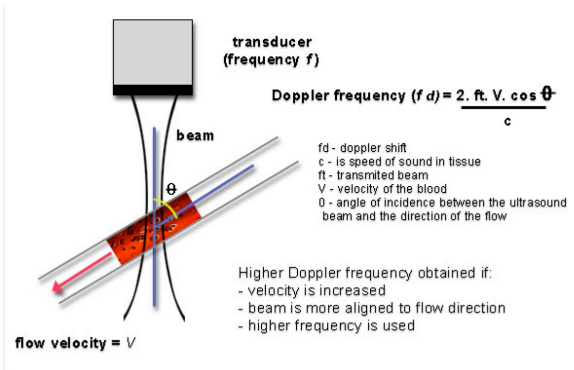
Prince and Links 2005

Doppler Effect



http://www.centrus.com.br/DiplomaFMF/SeriesFMF/doppler/capitulos-html/chapter_01.htm

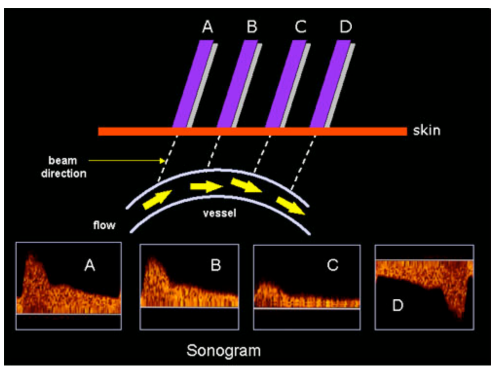
Doppler Effect

$$\Delta f = \frac{2vf_0}{c-v} \approx \frac{2vf_0}{c}$$

Example
v = 50 cm/s
c = 1500 m/s
f0 = 5 MHz

$$\frac{2vf_0}{c} = 3333\ Hz$$

Doppler



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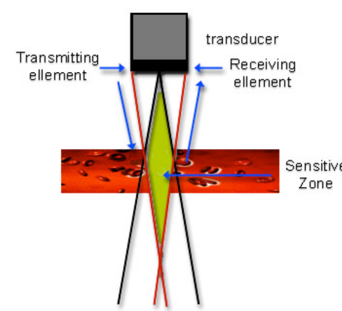
CW Doppler



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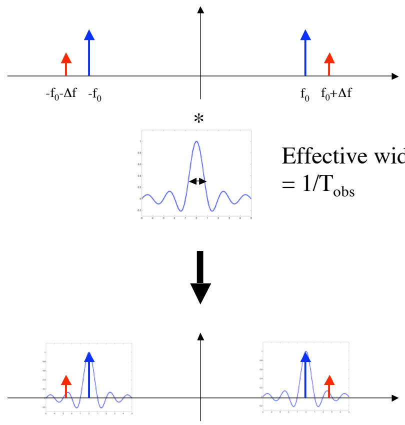
CW Doppler



Effective width = $1/T_{obs}$

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CW Doppler

Resolution $\Delta f = 1/T_{obs}$

$$\Delta v = \frac{c\Delta f}{2f_0} = \frac{c}{2T_{obs}f_0}$$

Example
Design goal: $\Delta v = 5\ cm/s;\ f_0 = 5\ MHz$

$$T_{obs} = \frac{c}{2\Delta v f_0} = \frac{1500m/s}{2(0.05m/s)(5\times 10^6)} = 3ms$$

Note that for a depth of 15 cm, it takes only 200 usec for echos to return.

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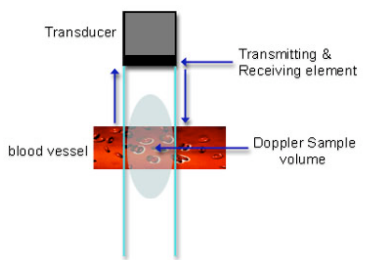
PW Doppler



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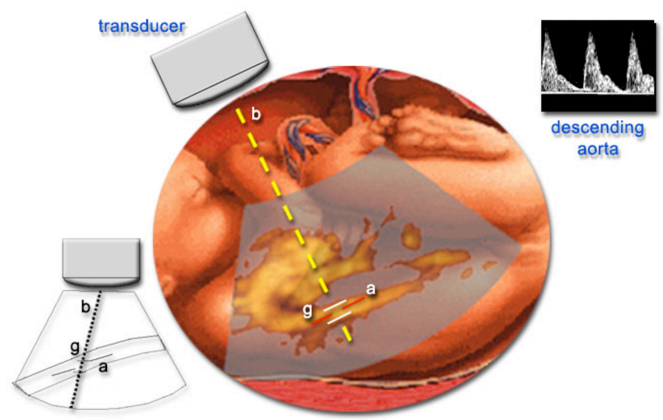
PW Doppler



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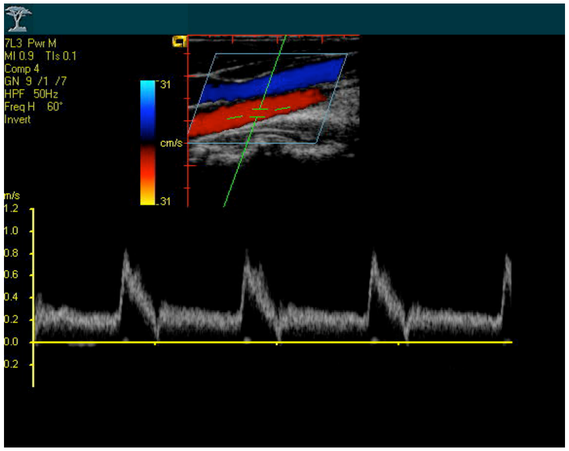


Siemens Medical Systems; jnormal common carotid artery

Aliasing

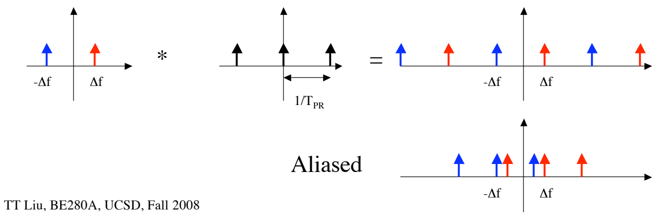
Measure Doppler shifts at a specified range

For unambiguous range, one pulse at a time.

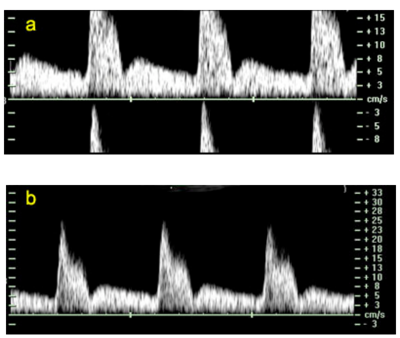
$T_{PR} = \frac{2r_{max}}{c}$ (e.g. 200 usec for 15 cm depth)

To avoid aliasing require

$\frac{1}{T_{PR}} > 2\Delta f_{max}$



Aliasing



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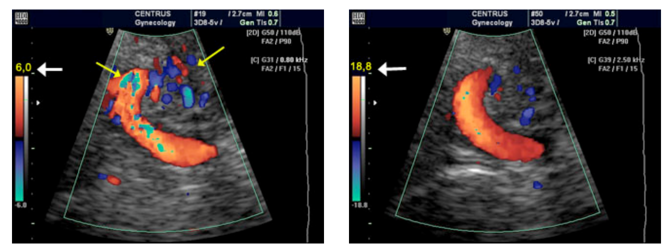
Aliasing



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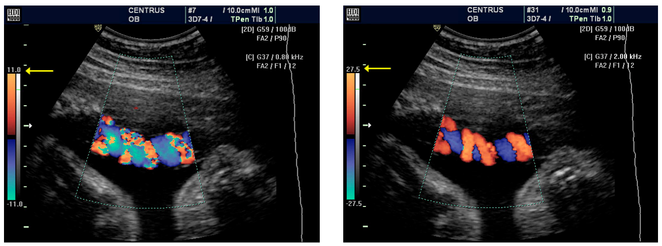
Aliasing



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PW Doppler

Velocity Resolution (same as with CW)

$$T_{obs} > \frac{1}{\Delta f} = \frac{c}{2\Delta v f_0}$$

Range Resolution

Want to interrogate velocities from a small region $\Delta z = \frac{cT_{pulse}}{2}$

We also need to make sure that particles remain within this region over the observation time T_{obs}

$$v_{max} T_{obs} < \Delta z \Rightarrow T_{obs} < \frac{\Delta z}{v_{max}} = \frac{cT_{pulse}}{2v_{max}}$$

PW Doppler

Design Example

$R_{max} = 6$ cm $\Rightarrow$ $T_{PR} = \frac{2(0.06m)}{1500m/s} = 80\ \mu\text{sec}$

$$\frac{1}{T_{PR}} > 2\Delta f_{max} = \frac{4v_{max} f_0}{c}$$

$\frac{c}{4T_{PR} f_0} > v_{max} \Rightarrow$ for $f_0 = 5MHz$ we find that $v_{max} < 93.75 cm/s$

If we choose $\Delta v = 1cm/s$ then $T_{obs} = \frac{c}{2\Delta v_{max} f_0} = 15ms$

Range resolution: $\Delta z > v_{max} T_{obs} = 1.4cm$

$$T_{pulse} = \frac{2\Delta z}{c} = 18.8u\text{sec}$$