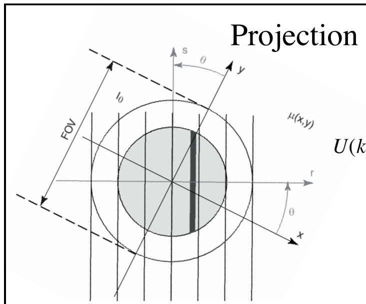
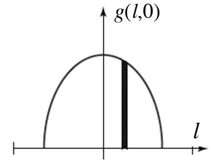
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2009
CT/Fourier Lecture 4

Projection Theorem



$$U(k_x,0)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu(x,y)e^{-j2\pi(k_x x+k_y y)}dxdy$$
$$=\int_{-\infty}^{-\infty}\left[\int_{-\infty}^{-\infty}\mu(x,y)dy\right]e^{-j2\pi k_x x}dx$$
$$=\int_{-\infty}^{-\infty}g(x,0)e^{-j2\pi k_x x}dx$$
$$=\int_{-\infty}^{-\infty}g(l,0)e^{-j2\pi kl}dl$$

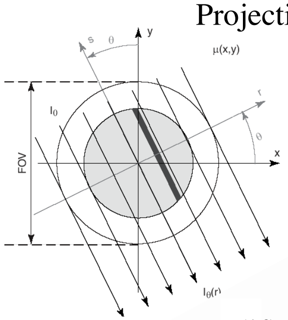


In-Class Example:

$$\mu(x,y)=\cos 2\pi x$$

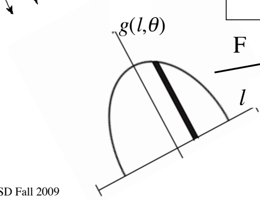
Suetens 2002

Projection Theorem



$$U(k_x,k_y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu(x,y)e^{-j2\pi(k_x x+k_y y)}dxdy$$
$$=F_{2D}\left[\mu(x,y)\right]$$

$$U(k_x,k_y)=G(k,\theta)$$
$$k_x=k\cos\theta$$
$$k_y=k\sin\theta$$
$$k=\sqrt{k_x^2+k_y^2}$$



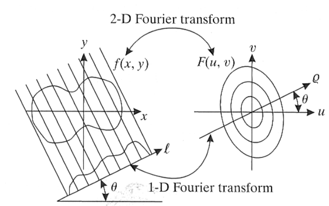
F

$$G(k,\theta)=\int_{-\infty}^{\infty}g(l,\theta)e^{-j2\pi kl}dl$$

Suetens 2002

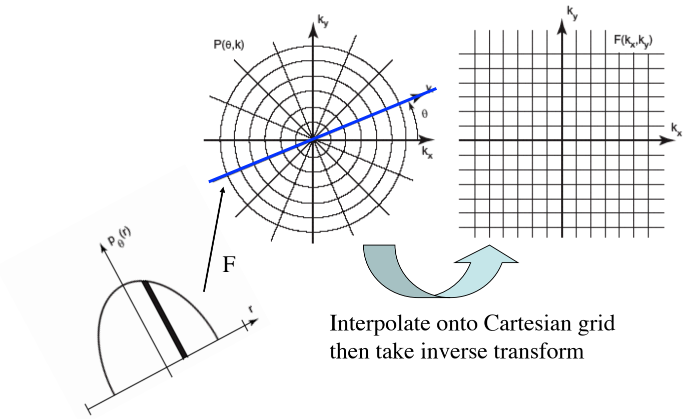
Projection Slice Theorem

$$G(\rho,\theta)=\int_{-\infty}^{\infty}g(l,\theta)e^{-j2\pi\rho l}dl$$
$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)\delta(x\cos\theta+y\sin\theta-l)e^{-j2\pi\rho l}dx\,dy\,dl$$
$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)e^{-j2\pi\rho(x\cos\theta+y\sin\theta)}dx\,dy$$
$$=F_{2D}\left[f(x,y)\right]_{u=\rho\cos\theta,v=\rho\sin\theta}$$



Prince&Links 2006

Fourier Reconstruction



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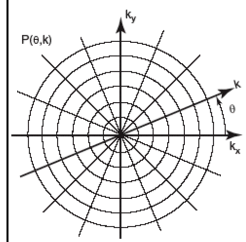
Polar Version of Inverse FT



$$\mu(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(k_x,k_y)e^{j2\pi(k_x x + k_y y)}dk_x dk_y$$

$$= \int_0^{2\pi}\int_0^{\infty} G(k,\theta)e^{j2\pi(xk\cos\theta + yk\sin\theta)}kdkd\theta$$

$$= \int_0^{\pi}\int_{-\infty}^{\infty} G(k,\theta)e^{j2\pi k(x\cos\theta + y\sin\theta)}|k|dkd\theta$$

Note :

$g(l,\theta+\pi) = g(-l,\theta)$

So

$G(k,\theta+\pi) = G(-k,\theta)$

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Filtered Backprojection

$$\mu(x,y) = \int_0^{\pi}\int_{-\infty}^{\infty} G(k,\theta)e^{j2\pi(xk\cos\theta + yk\sin\theta)}|k|dkd\theta$$

$$= \int_0^{\pi}\int_{-\infty}^{\infty} |k|G(k,\theta)e^{j2\pi kl}dkd\theta$$

$$= \int_0^{\pi} g^*(l,\theta)d\theta$$ ← Backproject a filtered projection

where $l = x\cos\theta + y\sin\theta$

$$g^*(l,\theta) = \int_{-\infty}^{\infty}|k|G(k,\theta)e^{j2\pi kl}dk$$

$$= g(l,\theta) * F^{-1}\left[|k|\right]$$

$$= g(l,\theta) * q(l)$$

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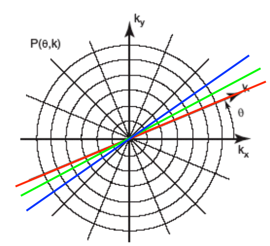
Fourier Interpretation



Density $\approx \dfrac{\text{N}}{\text{circumference}} \approx \dfrac{\text{N}}{2\pi|\text{k}|}$

Low frequencies are oversampled. So to compensate for this, multiply the k-space data by |k| before inverse transforming.

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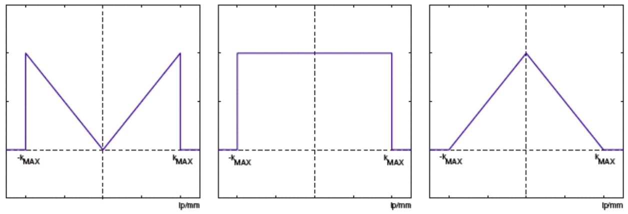
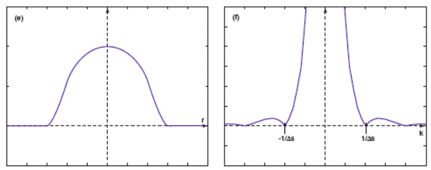
Kak and Slaney; Suetens 2002

Ram-Lak Filter



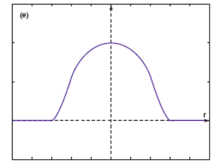
$k_{max}=1/\Delta s$



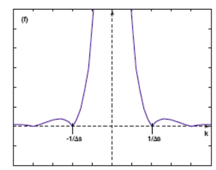
Suetens 2002

Reconstruction Path

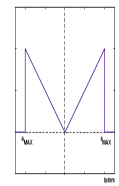
Projection



F ↓



x

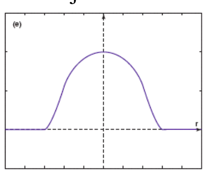


F^{-1} → Filtered Projection → Back-Project

Suetens 2002

Reconstruction Path

Projection



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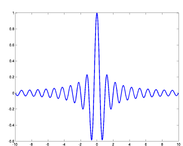


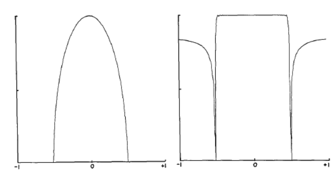
→ Filtered Projection

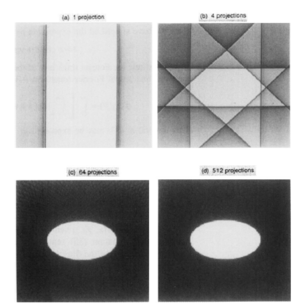
↓

Back-Project

Suetens 2002

Example





Kak and Slaney

Example

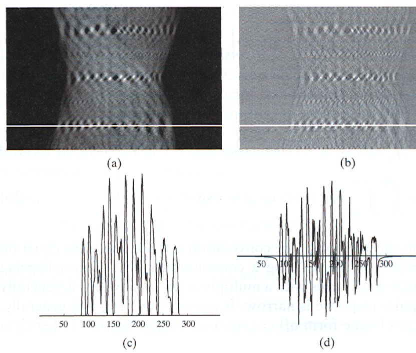


Figure 6.15
Convolution step:
(a) Original sinogram;
(b) filtered sinogram;
(c) profile of sinogram row [white line in (a)]; and
(d) profile of filtered sinogram row [white line in (b)].

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Prince and Links 2005

Example

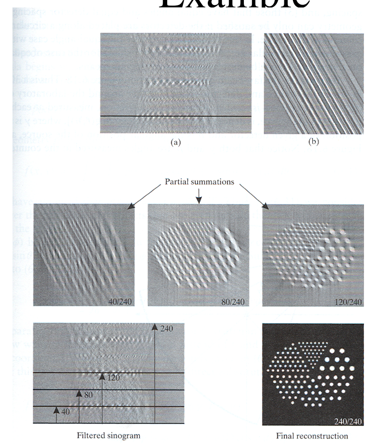

Figure 6.16
Backprojection step.

Figure 6.17
Summation step.

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Prince and Links 2005