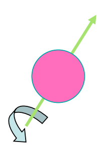
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2009
MRI Lecture 1

Spin

- Intrinsic angular momentum of elementary particles -- electrons, protons, neutrons.
- Spin is quantized. Key concept in Quantum Mechanics.

Magnetic Moment and Angular Momentum



A charged sphere spinning about its axis has angular momentum and a magnetic moment.

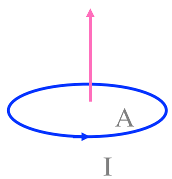
This is a classical analogy that is useful for understanding quantum spin, but remember that it is only an analogy!

Relation: $\boldsymbol{\mu} = \gamma\, \mathbf{S}$ where γ is the gyromagnetic ratio and $\mathbf{S}$ is the spin angular momentum.

Nuclear Spin Rules

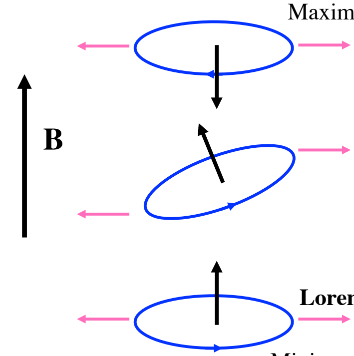
Number of Protons	Number of Neutrons	Spin	Examples
Even	Even	0	^{12}C, ^{16}O
Even	Odd	j/2	^{17}O
Odd	Even	j/2	^{1}H, ^{23}Na, ^{31}P
Odd	Odd	j	^{2}H

Classical Magnetic Moment



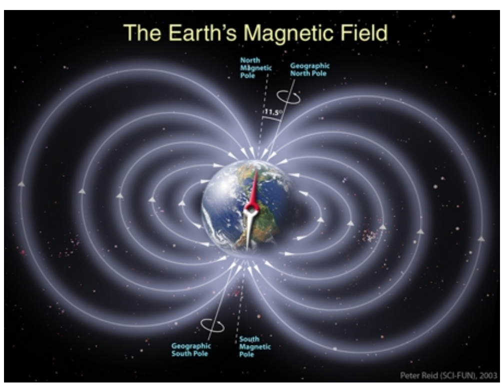
$$\vec{\mu} = IA\hat{n}$$

Energy in a Magnetic Field



$$E = -\vec{\mu} \bullet \mathbf{B} = -\mu_z B$$

Energy in a Magnetic Field



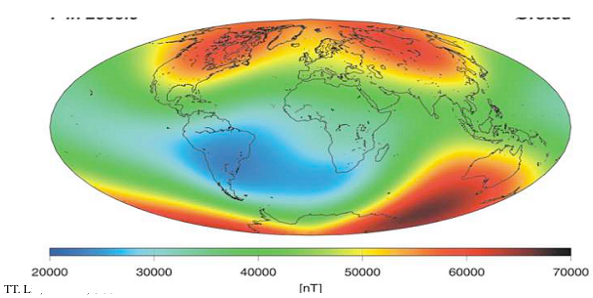
www.qi-whiz.com/images/ Earth-magnetic-field.jpg

Magnetic Field Units

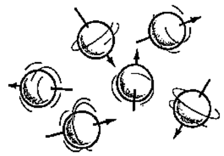
1 Tesla = 10,000 Gauss

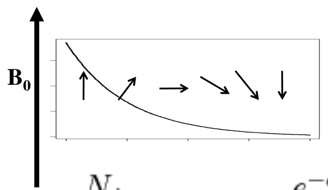
Earth's field is about 0.5 Gauss

0.5 Gauss = 0.5×10^{-4} T = 50 μT

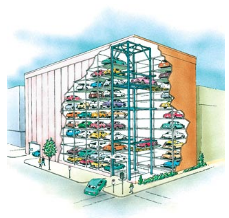


Boltzmann Distribution





$$\frac{N_j}{N} \equiv \mathcal{P}(\epsilon_j) = \frac{e^{-\epsilon_j/(kT)}}{Q}.$$



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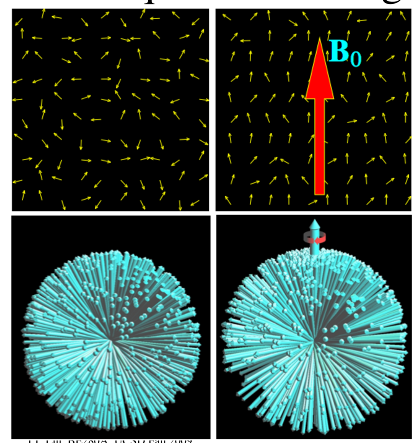
Equilibrium Magnetization



$$\mathbf{M}_0 = N\langle \mu_z \rangle$$
$$\approx N\mu_z^2 B/(kT)$$
$$= N\gamma^2\hbar^2 B/(4kT)$$

N = number of nuclear spins per unit volume
Magnetization is proportional to applied field.

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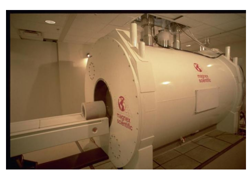
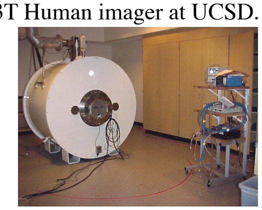
Bigger is better

3T Human imager at UCSD.



7T Human imager at U. Minn.



7T Rodent Imager at UCSD

9.4T Human imager at UIC

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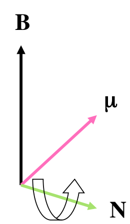
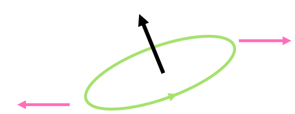
Torque



For a non-spinning magnetic moment, the torque will try to align the moment with magnetic field (e.g. compass needle)

N = μ x B

Torque



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Precession

Torque

$$\mathbf{N} = \mu \times \mathbf{B}$$

$$\frac{d\mathbf{S}}{dt} = \mathbf{N}$$

Change in Angular momentum

$$\frac{d\mathbf{S}}{dt} = \mu \times \mathbf{B}$$

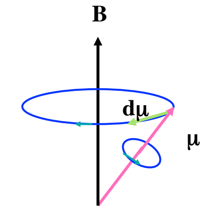
$$\mu = \gamma\, \mathbf{S}$$

Relation between magnetic moment and angular momentum

$$\frac{d\mu}{dt} = \mu \times \gamma\mathbf{B}$$

Precession

$$\frac{d\mu}{dt} = \mu \times \gamma\mathbf{B}$$

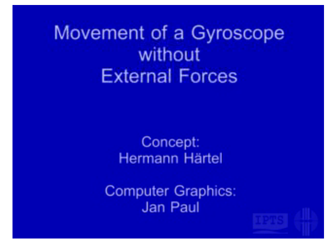


Analogous to motion of a gyroscope

Precesses at an angular frequency of

$$\omega = \gamma\, \mathbf{B}$$

This is known as the **Larmor** frequency.



http://www.astrophysik.uni-kiel.de/~hhaertelmpg_e/gyros_free.mpg

Magnetization Vector

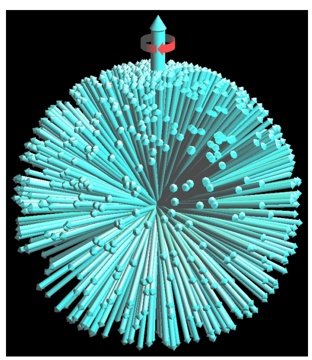
Vector sum of the magnetic moments over a volume.

For a sample at equilibrium in a magnetic field, the transverse components of the moments cancel out, so that there is only a longitudinal component.

Equation of motion is the same form as for individual moments.

$$\mathbf{M} = \frac{1}{V} \sum_{\substack{\text{protons} \\ \text{in V}}} \mu_i$$

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}$$



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Gyromagnetic Ratios

Nucleus	Spin	Magnetic Moment	$\gamma/(2\pi)$ (MHz/Tesla)	Abundance
^{1}H	1/2	2.793	42.58	88 M
^{23}Na	3/2	2.216	11.27	80 mM
^{31}P	1/2	1.131	17.25	75 mM

Source: Haacke et al., p. 27

Larmor Frequency

$\omega = \gamma B$ Angular frequency in rad/sec

$f = \gamma B / (2\pi)$ Frequency in cycles/sec or Hertz, Abbreviated Hz

For a 1.5 T system, the Larmor frequency is 63.86 MHz which is 63.86 million cycles per second. For comparison, KPBS-FM transmits at 89.5 MHz.

Note that the earth's magnetic field is about 50 μT, so that a 1.5T system is about 30,000 times stronger.

Notation and Units

1 Tesla = 10,000 Gauss

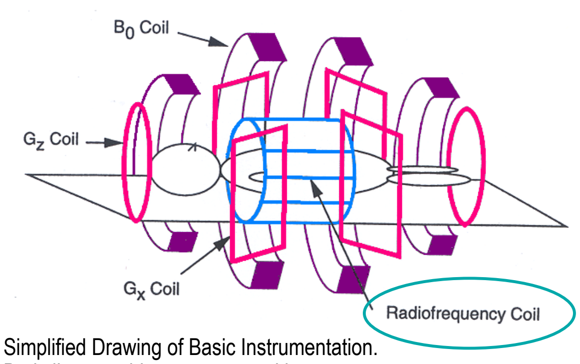
Earth's field is about 0.5 Gauss

0.5 Gauss = 0.5×10^{-4} T = 50 μT

$\gamma = 26752$ radians/second/Gauss

$\bar{\gamma} = \gamma / 2\pi = 4258$ Hz/Gauss

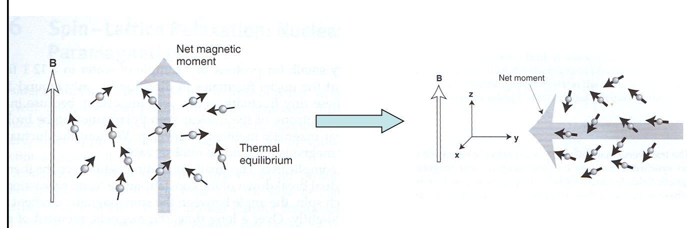
$= 42.58$ MHz/Tesla

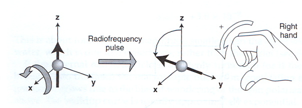


Simplified Drawing of Basic Instrumentation.
Body lies on table encompassed by
coils for static field B_o,
gradient fields (two of three shown),
and radiofrequency field B_1.

Image, caption: copyright Nishimura, Fig. 3.15

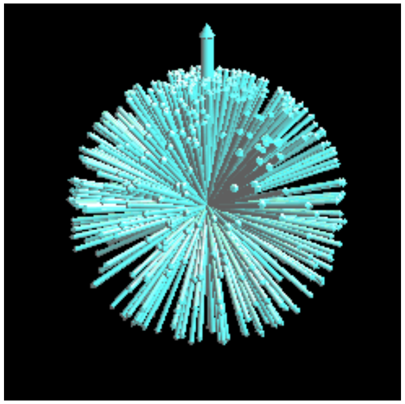
RF Excitation





From Levitt, Spin Dynamics, 2001

RF Excitation



http://www.drcmr.dk/main/content/view/213/74/

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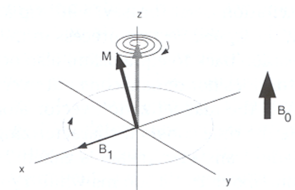


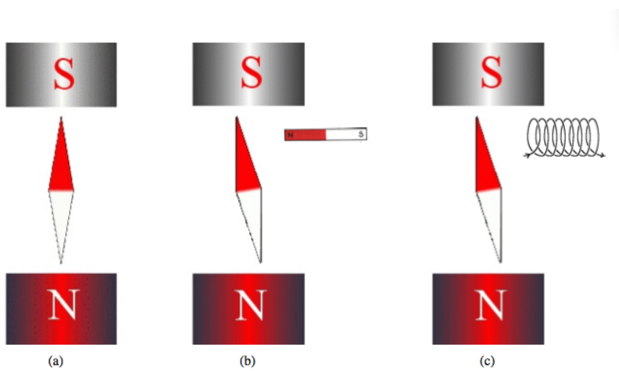
Image & caption: Nishimura, Fig. 3.2

At equilibrium, net magnetizaion is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

B_1 radiofrequency field tuned to Larmor frequency and applied in transverse (*xy*) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the *z*-axis.
- lab frame of reference

http://www.eecs.umich.edu/%7EdnollBME516/

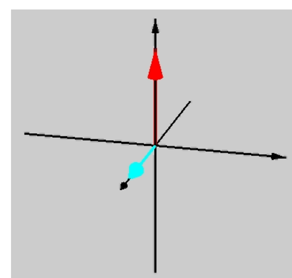
On-Resonance Excitation

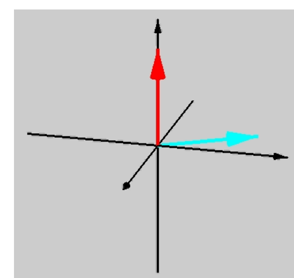


Hanson 2009

http://www.drcmr.dk/JavaCompass/

RF Excitation

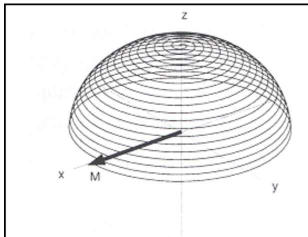


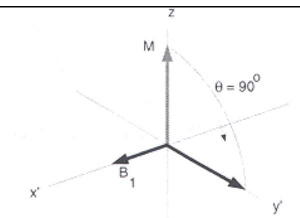


http://www.eecs.umich.edu/%7EdnollBME516/

Rotating Frame of Reference

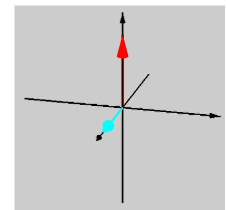
Reference everything to the magnetic field at isocenter.





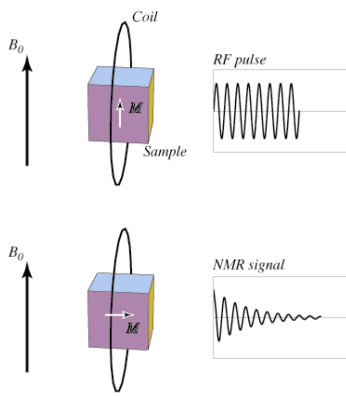
a) Laboratory frame behavior of **M**   b) Rotating frame behavior of **M**

Images & caption: Nishimura, Fig. 3.3



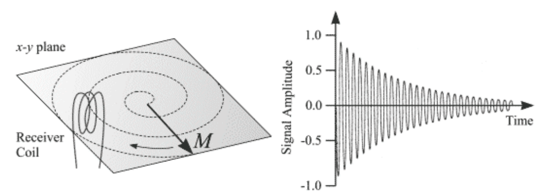
http://www.eecs.umich.edu/%7Ednoll/BME516/

RF Excitation



From Buxton 2002

Free Induction Decay (FID)



http://www.easymeasure.co.uk/principlesmri.aspx