

Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2009
MRI Lecture 1

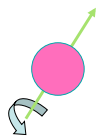
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Spin

- Intrinsic angular momentum of elementary particles -- electrons, protons, neutrons.
- Spin is quantized. Key concept in Quantum Mechanics.

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Magnetic Moment and Angular Momentum



A charged sphere spinning about its axis has angular momentum and a magnetic moment.

This is a classical analogy that is useful for understanding quantum spin, but remember that it is only an analogy!

Relation: $\boldsymbol{\mu} = \gamma \mathbf{S}$ where γ is the gyromagnetic ratio and \mathbf{S} is the spin angular momentum.

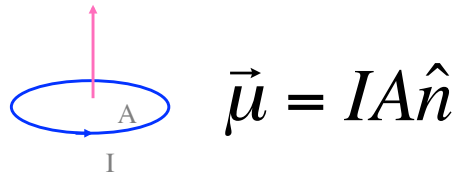
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Nuclear Spin Rules

Number of Protons	Number of Neutrons	Spin	Examples
Even	Even	0	^{12}C , ^{16}O
Even	Odd	$j/2$	^{17}O
Odd	Even	$j/2$	^1H , ^{23}Na , ^{31}P
Odd	Odd	j	^2H

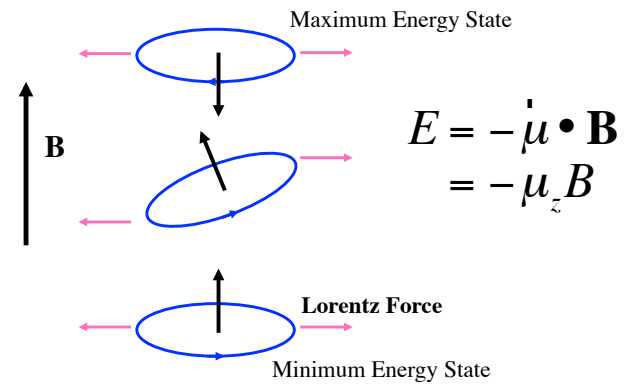
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Classical Magnetic Moment



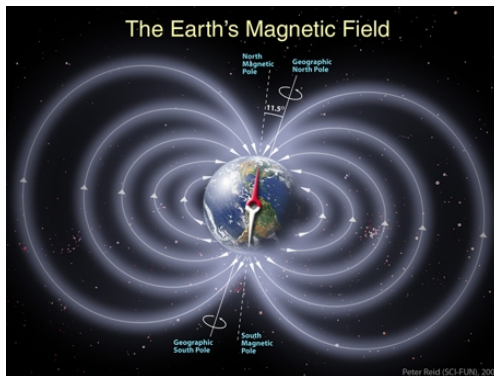
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Energy in a Magnetic Field



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Energy in a Magnetic Field



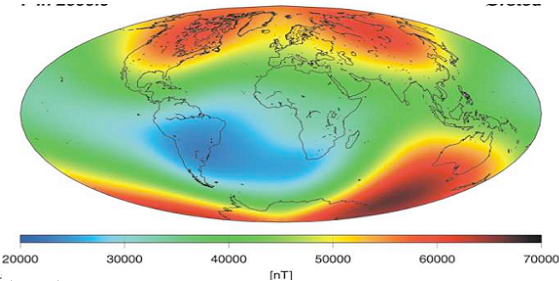
TT. Liu, BE280A, UCSD Fall 2009 www.qi-whiz.com/images/Earth-magnetic-field.jpg

Magnetic Field Units

1 Tesla = 10,000 Gauss

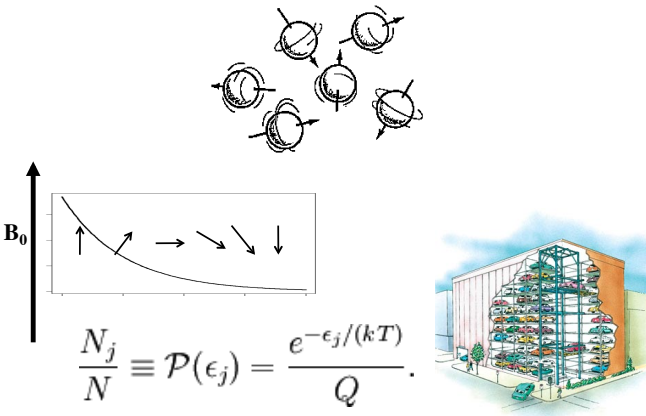
Earth's field is about 0.5 Gauss

0.5 Gauss = $0.5 \times 10^{-4} \text{ T} = 50 \mu\text{T}$



TT. L.

Boltzmann Distribution

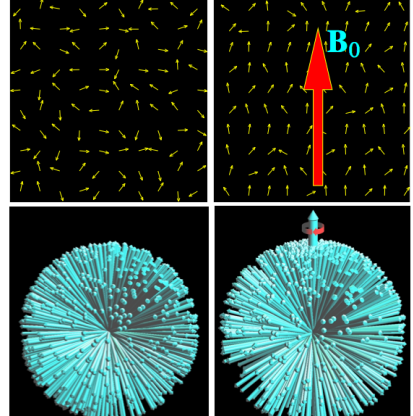


The diagram illustrates the Boltzmann distribution of magnetic moments. At the top, several magnetic moments are shown as circles with arrows pointing in various directions. Below this, a graph plots the magnetic field B_0 on the vertical axis against the probability of a magnetic moment being in a certain state. The curve shows that as the magnetic field increases, the probability of finding moments in the lower energy state (pointing up) increases, while the probability of finding them in the higher energy state (pointing down) decreases. The Boltzmann distribution equation is given as:

$$\frac{N_j}{N} \equiv \mathcal{P}(\epsilon_j) = \frac{e^{-\epsilon_j/(kT)}}{Q}$$

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Equilibrium Magnetization



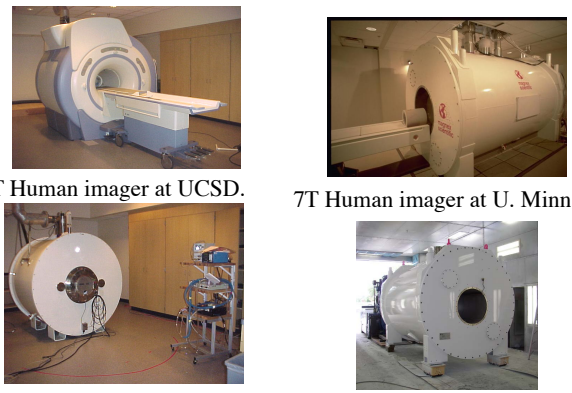
The diagram shows four panels illustrating equilibrium magnetization. The top-left panel shows random magnetic moments (yellow arrows) on a black background. The top-right panel shows the moments aligned with an external magnetic field B_0 (red arrow). The bottom-left panel shows a 3D representation of the moments as cyan arrows. The bottom-right panel shows the moments aligned with B_0 in a 3D representation. The magnetization equations are:

$$\begin{aligned} \mathbf{M}_0 &= N \langle \mu_z \rangle \\ &\approx N \mu_z^2 B / (kT) \\ &= N \gamma^2 \hbar^2 B / (4kT) \end{aligned}$$

N = number of nuclear spins per unit volume
Magnetization is proportional to applied field.

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Bigger is better

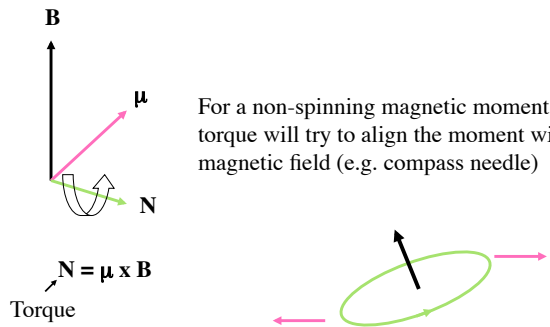


The images show four MRI scanners of increasing size and field strength:

- 3T Human imager at UCSD.
- 7T Human imager at U. Minn.
- 7T Rodent Imager at UCSD.
- 9.4T Human imager at UIC.

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Torque



The diagram shows a magnetic moment μ (pink arrow) and a magnetic field B (black arrow). The torque N (green arrow) is shown as a curved arrow around the axis of B . The equation for torque is:

$$\mathbf{N} = \boldsymbol{\mu} \times \mathbf{B}$$

Torque

For a non-spinning magnetic moment, the torque will try to align the moment with magnetic field (e.g. compass needle)

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Precession

Torque

$$\mathbf{N} = \boldsymbol{\mu} \times \mathbf{B}$$

$$\frac{d\mathbf{S}}{dt} = \mathbf{N}$$

Change in Angular momentum

$$\frac{d\mathbf{S}}{dt} = \boldsymbol{\mu} \times \mathbf{B}$$

}

$$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\mu} \times \gamma \mathbf{B}$$

$$\boldsymbol{\mu} = \gamma \mathbf{S}$$

Relation between magnetic moment and angular momentum

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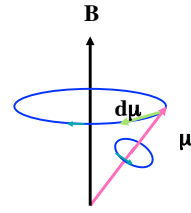
Precession

Analogous to motion of a gyroscope

Precesses at an angular frequency of

$\omega = \gamma \mathbf{B}$

This is known as the **Larmor** frequency.



Movement of a Gyroscope without External Forces

Concept: Hermann Härtel
Computer Graphics: Jan Paul

http://www.astrophysik.uni-kiel.de/~hhaertelmpg_e/gyros_free.mpg

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Magnetization Vector

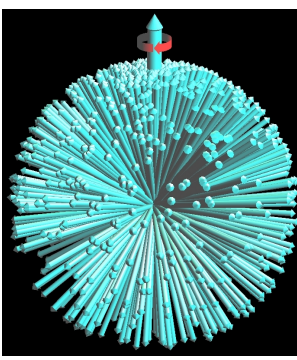
Vector sum of the magnetic moments over a volume.

For a sample at equilibrium in a magnetic field, the transverse components of the moments cancel out, so that there is only a longitudinal component.

Equation of motion is the same form as for individual moments.

$$\mathbf{M} = \frac{1}{V} \sum_{\text{protons in } V} \boldsymbol{\mu}_i$$

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}$$



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Gyromagnetic Ratios

Nucleus	Spin	Magnetic Moment	$\gamma/(2\pi)$ (MHz/Tesla)	Abundance
¹ H	1/2	2.793	42.58	88 M
²³ Na	3/2	2.216	11.27	80 mM
³¹ P	1/2	1.131	17.25	75 mM

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Larmor Frequency

$\omega = \gamma \mathbf{B}$ Angular frequency in rad/sec

$f = \gamma \mathbf{B} / (2\pi)$ Frequency in cycles/sec or Hertz, Abbreviated Hz

For a 1.5 T system, the Larmor frequency is 63.86 MHz which is 63.86 million cycles per second. For comparison, KPBS-FM transmits at 89.5 MHz.

Note that the earth's magnetic field is about 50 μT , so that a 1.5T system is about 30,000 times stronger.

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Notation and Units

1 Tesla = 10,000 Gauss

Earth's field is about 0.5 Gauss

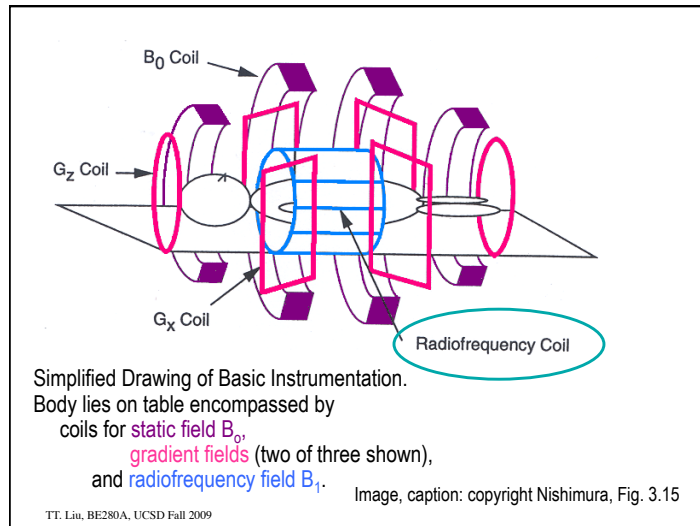
0.5 Gauss = $0.5 \times 10^{-4} \text{ T} = 50 \mu\text{T}$

$\gamma = 26752 \text{ radians/second/Gauss}$

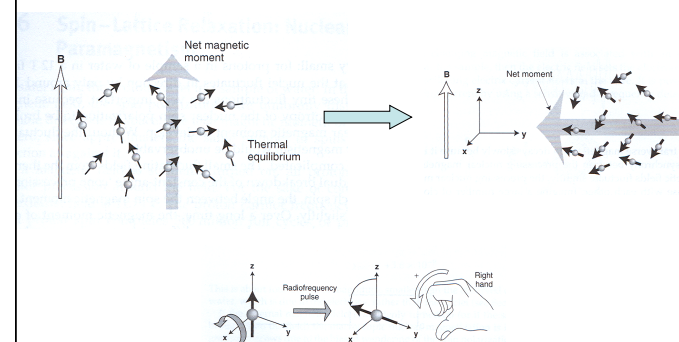
$\gamma = \gamma / 2\pi = 4258 \text{ Hz/Gauss}$

= 42.58 MHz/Tesla

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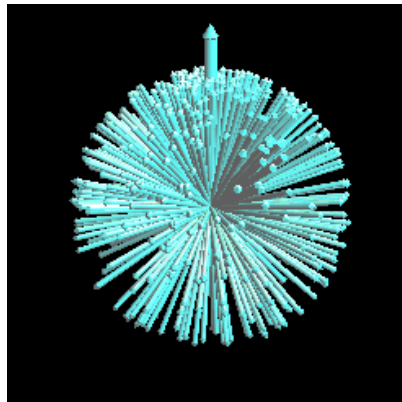
RF Excitation



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From Levitt, Spin Dynamics, 2001

RF Excitation



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<http://www.drcmr.dk/main/content/view/213/74/>

RF Excitation

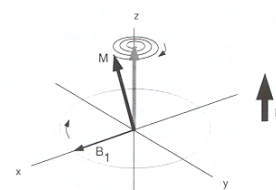


Image & caption: Nishimura, Fig. 3.2

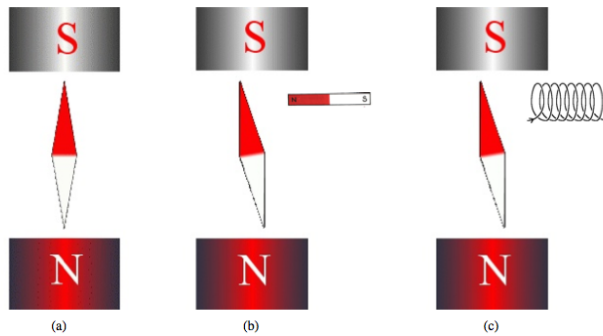
At equilibrium, net magnetization is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

B_1 radiofrequency field tuned to Larmor frequency and applied in transverse (xy) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the z -axis.
- lab frame of reference

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<http://www.eecs.umich.edu/~7EdnoIHBME516/>

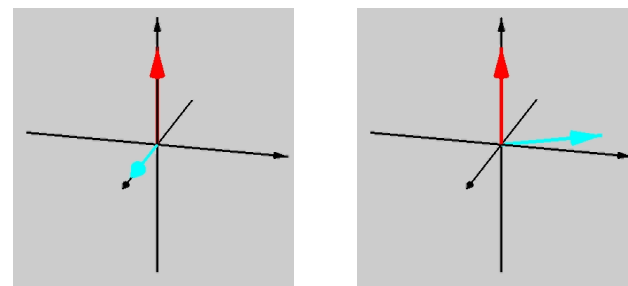
On-Resonance Excitation



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Hanson 2009
<http://www.drcmr.dk/JavaCompass/>

RF Excitation



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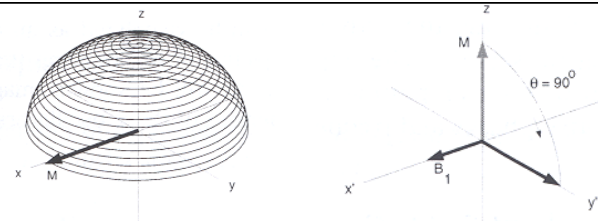
<http://www.eecs.umich.edu/~7EdnoIHBME516/>

Rotating Frame of Reference

Reference everything to the magnetic field at isocenter.

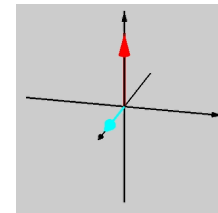


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a) Laboratory frame behavior of **M**
Images & caption: Nishimura, Fig. 3.3

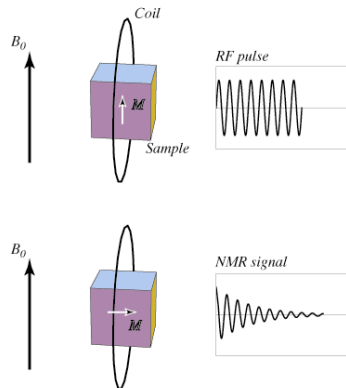
b) Rotating frame behavior of **M**



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<http://www.eecs.umich.edu/~7Edno/BME516/>

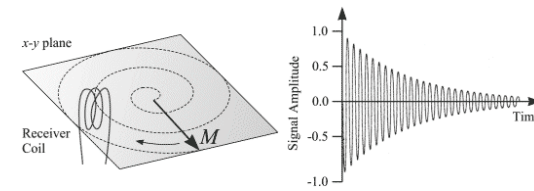
RF Excitation



From Buxton 2002

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Free Induction Decay (FID)



<http://www.easymeasure.co.uk/principlesmri.aspx>

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