Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2009
MRI Lecture 2

Relaxation

An excitation pulse rotates the magnetization vector away from its equilibrium state (purely longitudinal). The resulting vector has both longitudinal $\mathbf{M_z}$ and tranverse $\mathbf{M_{xy}}$ components.

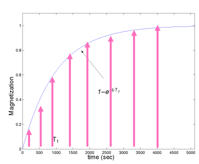
Due to thermal interactions, the magnetization will return to its equilibrium state with characteristic time constants.

T_1 spin-lattice time constant, return to equilibrium of $\mathbf{M_z}$

T_2 spin-spin time constant, return to equilibrium of $\mathbf{M_{xy}}$

Longitudinal Relaxation

$$\frac{d\mathbf{M}_z}{dt} = -\frac{M_z - M_0}{T_1}$$

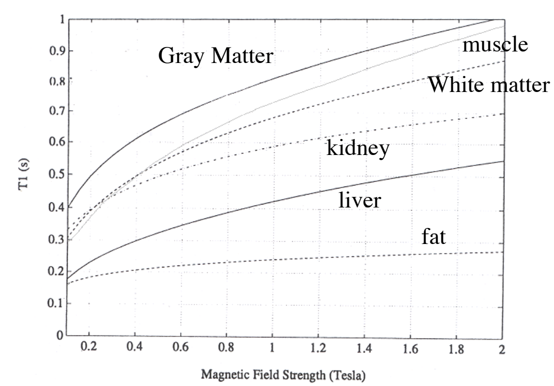


After a 90 degree pulse $M_z(t) = M_0(1 - e^{-t/T_1})$

Due to exchange of energy between nuclei and the lattice (thermal vibrations). Process continues until thermal equilibrium as determined by Boltzmann statistics is obtained.

The energy ΔE required for transitions between down to up spins, increases with field strength, so that T_1 increases with $\mathbf{B}$.

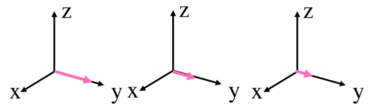
T1 Values



Image, caption: Nishimura, Fig. 4.2

Transverse Relaxation

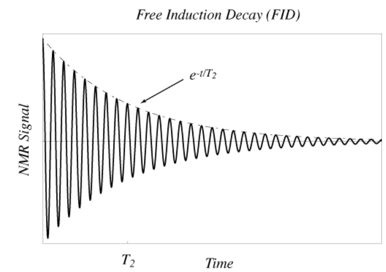
$$\frac{d\mathbf{M}_{xy}}{dt} = -\frac{M_{xy}}{T_2}$$



Each spin's local field is affected by the z-component of the field due to other spins. Thus, the Larmor frequency of each spin will be slightly different. This leads to a dephasing of the transverse magnetization, which is characterized by an exponential decay.

T_2 is largely independent of field. T_2 is short for low frequency fluctuations, such as those associated with slowly tumbling macromolecules.

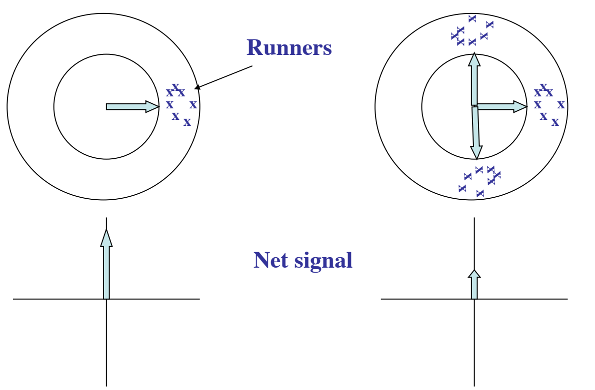
T2 Relaxation



After a 90 degree excitation

$$M_{xy}(t) = M_0 e^{-t/T_2}$$

T2 Relaxation



Credit: Larry Frank

T2 Values

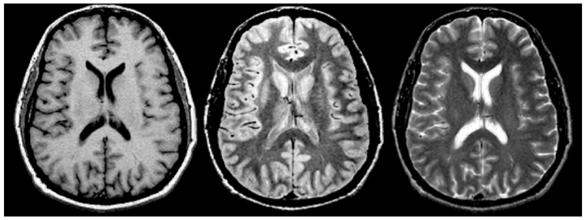
Tissue	T_2 (ms)
gray matter	100
white matter	92
muscle	47
fat	85
kidney	58
liver	43
CSF	4000

Table: adapted from Nishimura, Table 4.2

Solids exhibit very short T_2 relaxation times because there are many low frequency interactions between the immobile spins.

On the other hand, liquids show relatively long T_2 values, because the spins are highly mobile and net fields average out.

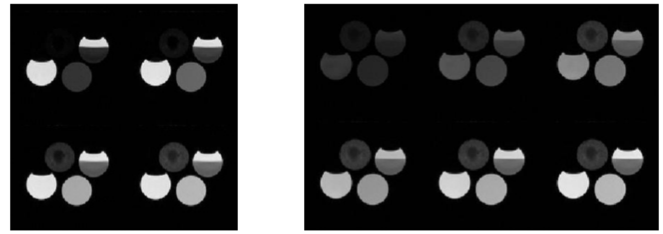
Example



T_1-weighted   Density-weighted T_2-weighted

Questions: How can one achieve T2 weighting? What are the relative T2's of the various tissues?

Example



(a) Four images, all obtained with a common TR=5 seconds and TE=90, 50, 20, 15 ms (shown in reading order).

(b) Six images obtained with a common TE=15 ms and TR=500, 1000, 2000, 3000, 4000, 5000 ms (shown in reading order).

Figure 8: Phantom data which illustrates signal intensity and contrast for bottles filled with jello af varying consistency. Where is T_1 long/short? How long, how short? The same for T_2? Which bottles might be pure water? Which jello is most firm? What pictures are the most T_1-, T_2- and PD-weighted?

Hanson 2009

Bloch Equation

$$\frac{d\mathbf{M}}{dt} = \underbrace{\mathbf{M}\times\gamma\mathbf{B}}_{\text{Precession}} - \underbrace{\frac{M_x\mathbf{i} + M_y\mathbf{j}}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_z - M_0)\mathbf{k}}{T_1}}_{\text{Longitudinal Relaxation}}$$

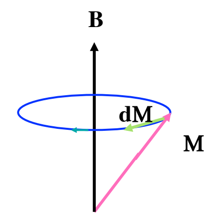
i, j, k are unit vectors in the x,y,z directions.

Free precession about static field

$$\frac{d\mathbf{M}}{dt} = \mathbf{M}\times\gamma\mathbf{B}$$

$$= \gamma\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \gamma\begin{pmatrix} \hat{i}(B_z M_y - B_y M_z) \\ -\hat{j}(B_z M_x - B_x M_z) \\ \hat{k}(B_y M_x - B_x M_y) \end{pmatrix}$$



Free precession about static field

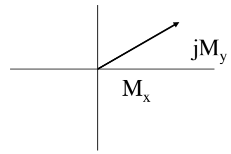
$$\begin{bmatrix} dM_x/dt \\ dM_y/dt \\ dM_z/dt \end{bmatrix} = \gamma \begin{bmatrix} B_z M_y - B_y M_z \\ B_x M_z - B_z M_x \\ B_y M_x - B_x M_y \end{bmatrix}$$

$$= \gamma \begin{bmatrix} 0 & B_z & -B_y \\ -B_z & 0 & B_x \\ B_y & -B_x & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

Precession

$$\begin{bmatrix} dM_x/dt \\ dM_y/dt \\ dM_z/dt \end{bmatrix} = \gamma \begin{bmatrix} 0 & B_0 & 0 \\ -B_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

Useful to define $M \equiv M_x + jM_y$



$$dM/dt = d/dt\left(M_x + iM_y\right)$$
$$= -j\gamma B_0 M$$

Solution is a time-varying phasor

$$M(t) = M(0)e^{-j\gamma B_0 t} = M(0)e^{-j\omega_0 t}$$

Question: which way does this rotate with time?

Matrix Form with B=B$_0$

$$\begin{bmatrix} dM_x/dt \\ dM_y/dt \\ dM_z/dt \end{bmatrix} = \begin{bmatrix} -1/T_2 & \gamma B_0 & 0 \\ -\gamma B_0 & 1/T_2 & 0 \\ 0 & 0 & -1/T_1 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ M_0/T_1 \end{bmatrix}$$

Z-component solution

$$M_z(t) = M_0 + (M_z(0) - M_0)e^{-t/T_1}$$

Saturation Recovery

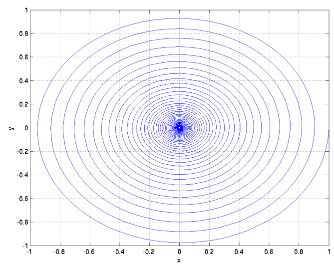
If $M_z(0) = 0$ then $M_z(t) = M_0(1 - e^{-t/T_1})$

Inversion Recovery

If $M_z(0) = -M_0$ then $M_z(t) = M_0(1 - 2e^{-t/T_1})$

Transverse Component

$M \equiv M_x + jM_y$

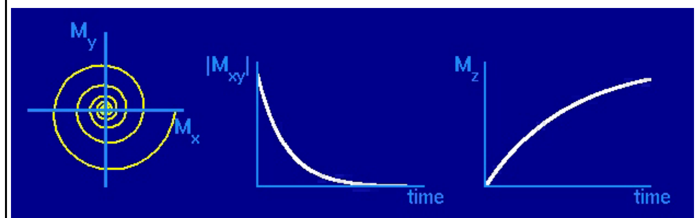


$$dM/dt = d/dt(M_x + iM_y)$$
$$= -j(\omega_0 + 1/T_2)M$$

$$M(t) = M(0)e^{-j\omega_0 t}e^{-t/T_2}$$

Summary

1) Longitudinal component recovers exponentially.
2) Transverse component precesses and decays exponentially.



Source: http://mrsrl.stanford.edu/~brian/mri-movies/

Summary

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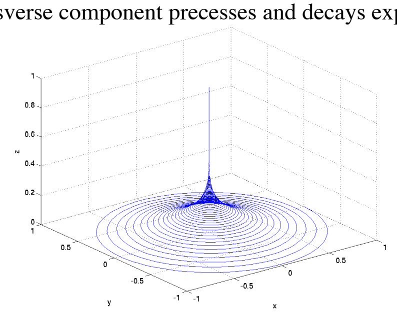


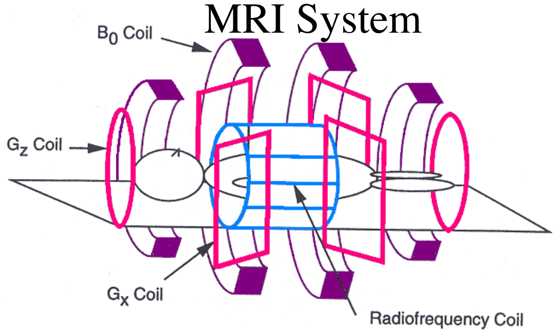
Fact: Can show that $T_2 < T_1$ in order for $|M(t)| \leq M_0$
Physically, the mechanisms that give rise to T_1 relaxation also contribute to transverse T_2 relaxation.

Gradients

Spins precess at the Larmor frequency, which is proportional to the local magnetic field. In a constant magnetic field $B_z=B_0$, all the spins precess at the same frequency (ignoring chemical shift).

Gradient coils are used to add a spatial variation to B_z such that $B_z(x,y,z) = B_0 + \Delta B_z(x,y,z)$. Thus, spins at different physical locations will precess at different frequencies.

MRI System



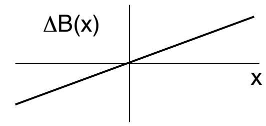
Simplified Drawing of Basic Instrumentation.
Body lies on table encompassed by
coils for static field B_o,
gradient fields (two of three shown),
and radiofrequency field B_1.

Image, caption: copyright Nishimura, Fig. 3.15

Imaging: localizing the NMR signal

RF and Gradient Coils

The local precession frequency can be changed in a position-dependent way by applying linear field gradients



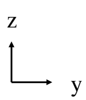
Resonant Frequency:

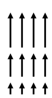
$\nu(x) = \gamma B_0 + \gamma \Delta B(x)$

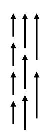
Credit: R. Buxton

Gradient Fields

$$B_z(x,y,z) = B_0 + \frac{\partial B_z}{\partial x}x + \frac{\partial B_z}{\partial y}y + \frac{\partial B_z}{\partial z}z$$
$$= B_0 + G_x x + G_y y + G_z z$$

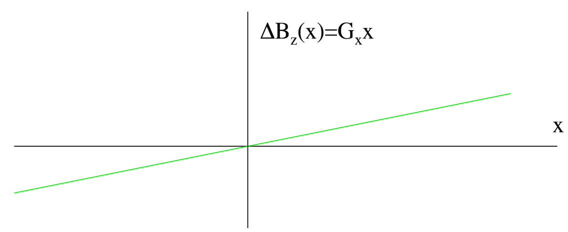






$$G_z = \frac{\partial B_z}{\partial z} > 0 \qquad G_y = \frac{\partial B_z}{\partial y} > 0$$

Interpretation



Spins Precess at $\gamma B_0 - \gamma G_x x$ (slower)

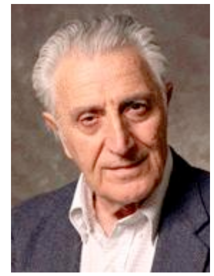
Spins Precess at γB_0

Spins Precess at at $\gamma B_0 + \gamma G_x x$ (faster)

Rotating Frame of Reference

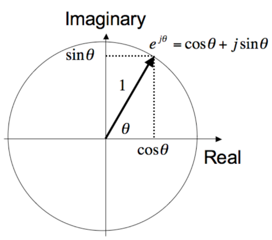
Reference everything to the magnetic field at isocenter.

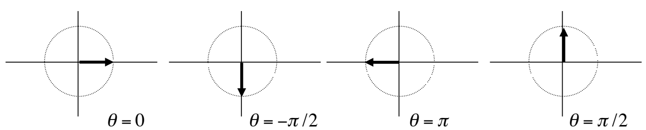
Spins



There is nothing that nuclear spins will not do for you, as long as you treat them as human beings.
Erwin Hahn

Phasors

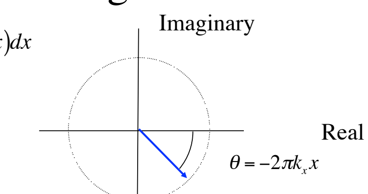




Phasor Diagram

$$G(k_x) = \int_{-\infty}^{\infty} g(x)\exp(-j2\pi k_x x)dx$$

$\theta = -2\pi k_x x$

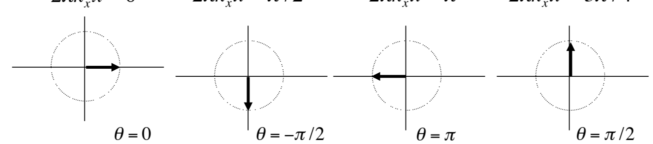


$k_x = 1;\ x = 0$ — $2\pi k_x x = 0$

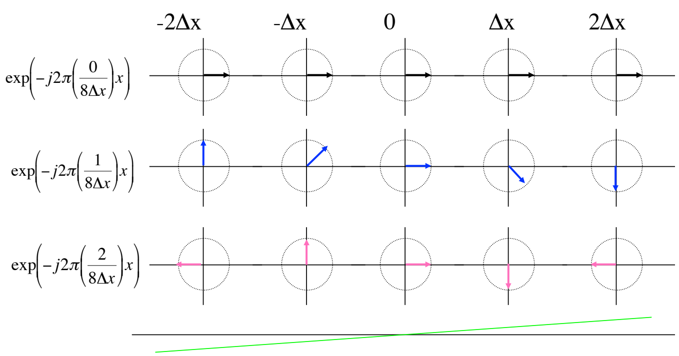
$x = 1/4$ — $2\pi k_x x = \pi/2$

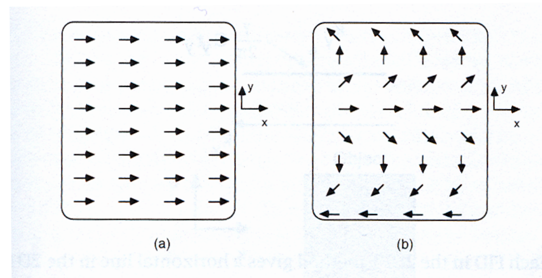
$x = 1/2$ — $2\pi k_x x = \pi$

$x = 3/2$ — $2\pi k_x x = 3\pi/4$



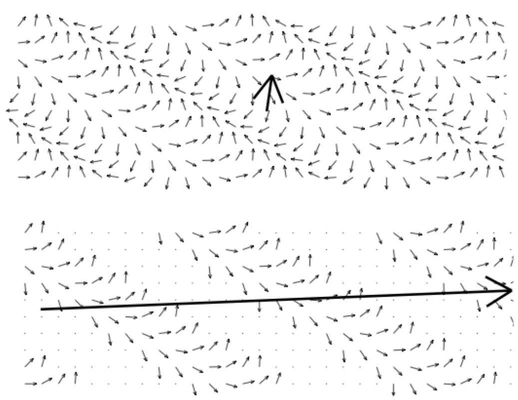
Interpretation





$k_x=0;\ k_y=0$ $\qquad$ $k_x=0;\ k_y\neq 0$

Fig 3.12 from Nishimura



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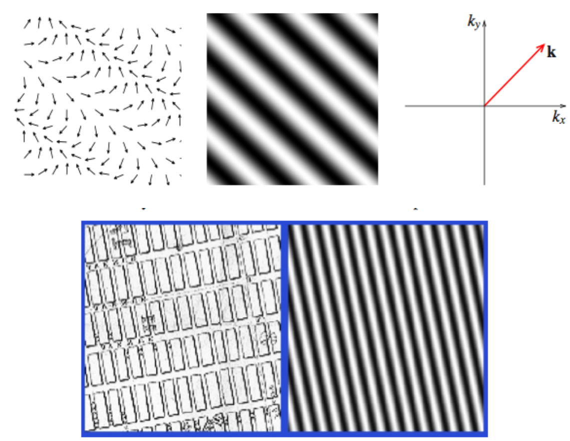
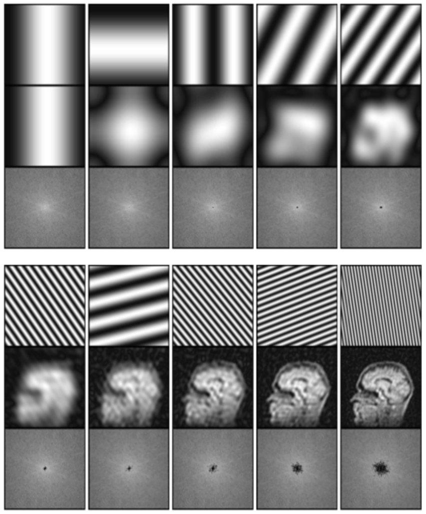
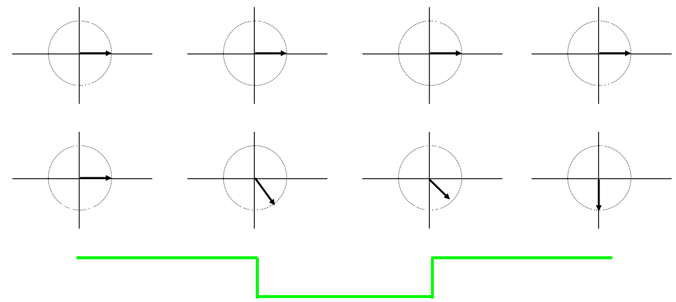


Hanson 2009

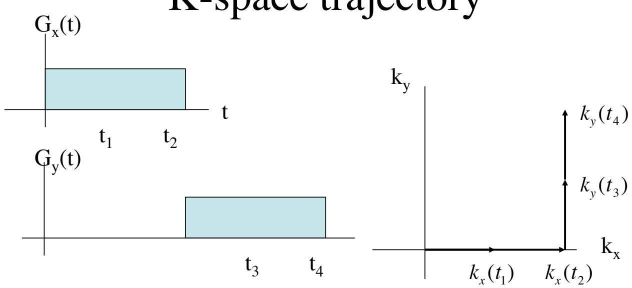


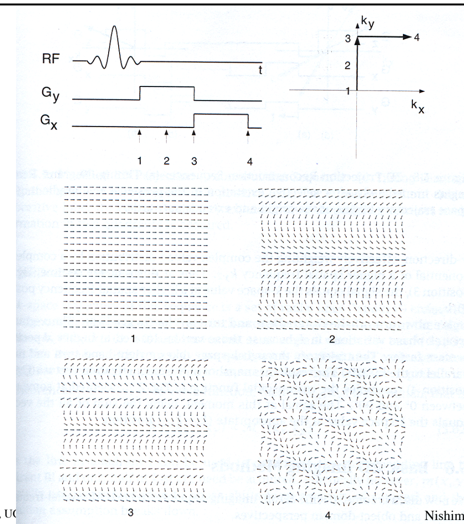
Hanson 2009

Phase with time-varying gradient



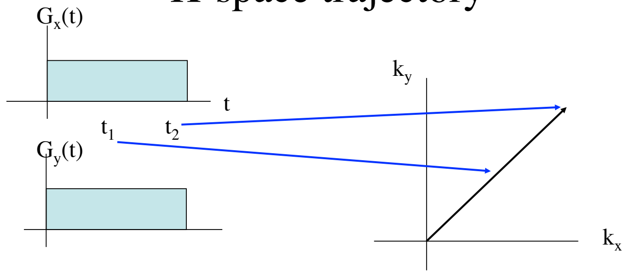
K-space trajectory





Nishimura 1996

K-space trajectory



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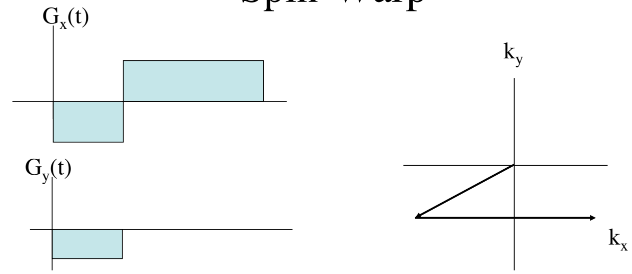
Spin-Warp



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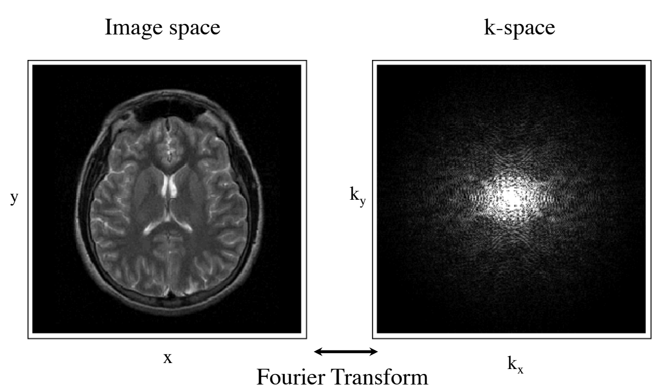
k-space



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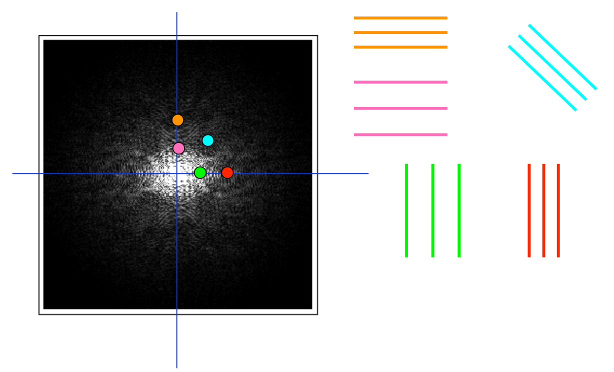

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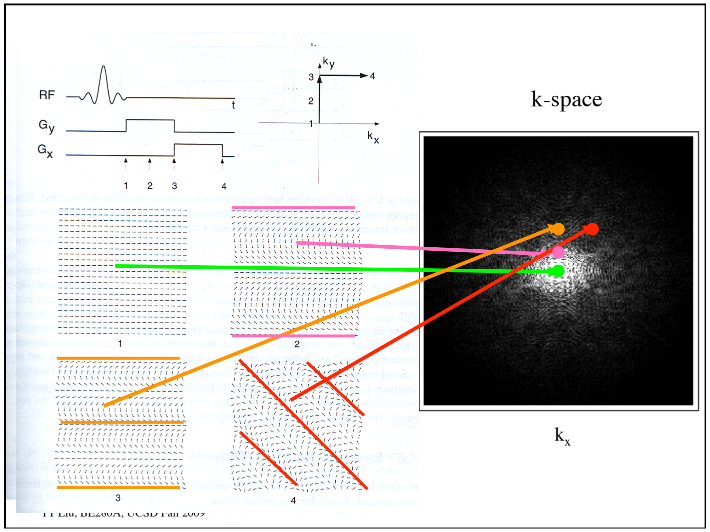
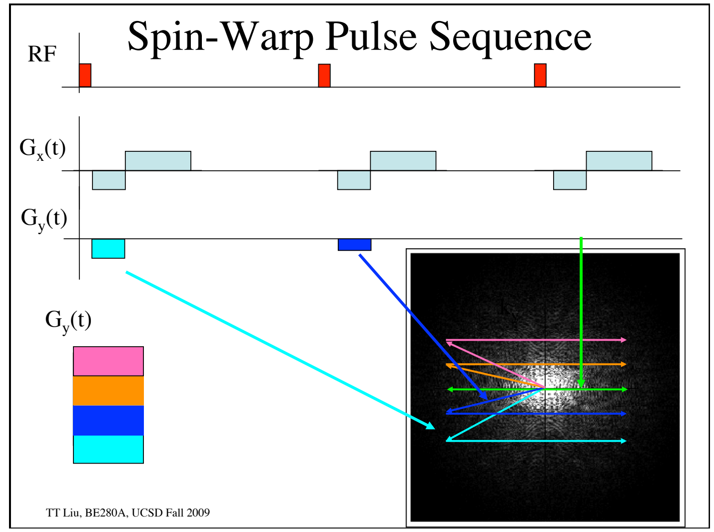

RF
Gy
Gx
1 2 3 4
ky
kx
k-space
kx


Spin-Warp Pulse Sequence
RF
Gx(t)
Gy(t)
Gy(t)
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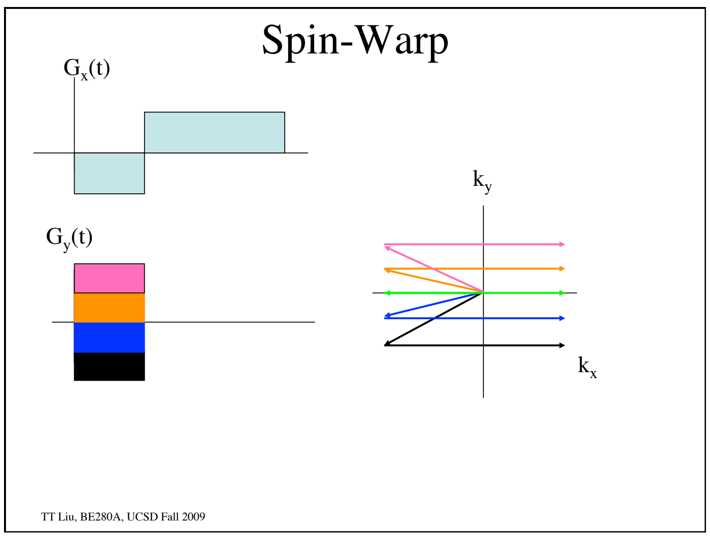
Spin-Warp
Gx(t)
Gy(t)
ky
kx
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