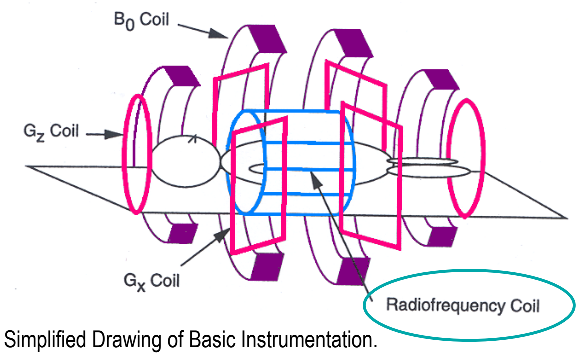
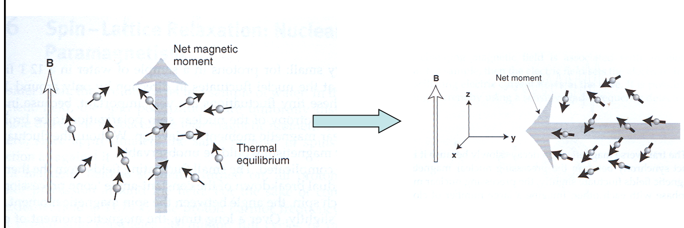
Bioengineering 280A
Principles of Biomedical Imaging

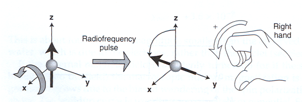
Fall Quarter 2009
MRI Lecture 4



Simplified Drawing of Basic Instrumentation.
Body lies on table encompassed by
coils for static field B_o,
gradient fields (two of three shown),
and radiofrequency field B_1.

RF Excitation





From Levitt, Spin Dynamics, 2001

RF Excitation

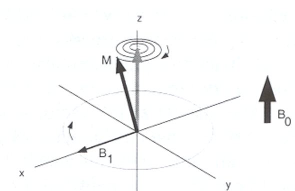
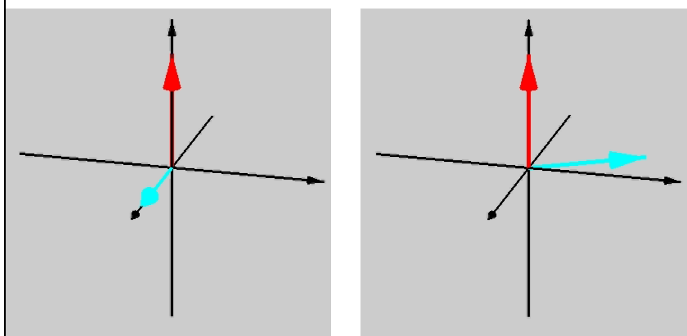


Image & caption: Nishimura, Fig. 3.2

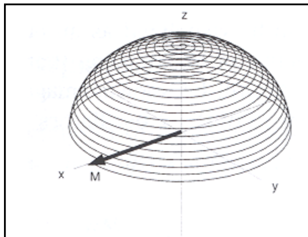
At equilibrium, net magnetizaion is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

B_1 radiofrequency field tuned to Larmor frequency and applied in transverse (*xy*) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the *z*-axis.
- lab frame of reference

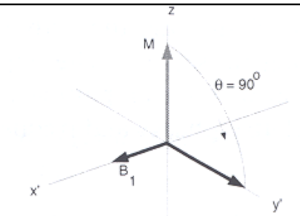
RF Excitation



http://www.eecs.umich.edu/%7EdnollBME516/

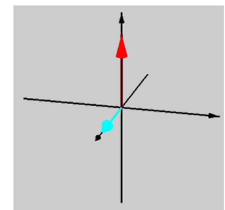


a) Laboratory frame behavior of **M**

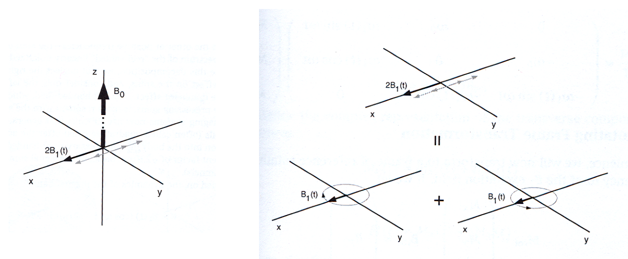


b) Rotating frame behavior of **M**

Images & caption: Nishimura, Fig. 3.3

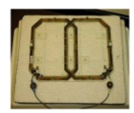


http://www.eecs.umich.edu/%7EdnollBME516/



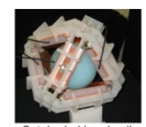
$$\mathbf{B}_1(t) = 2B_1(t)\cos(\omega t)\mathbf{i}$$
$$= B_1(t)\left(\cos(\omega t)\mathbf{i} - \sin(\omega t)\mathbf{j}\right) + B_1(t)\left(\cos(\omega t)\mathbf{i} + \sin(\omega t)\mathbf{j}\right)$$

Nishimura 1996



2-channel phased array for cardio MRI (Rx)

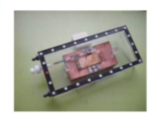
Breast coil with detached coupling box



Octahedral head coil with 12 TxRx channels

Cable head coil (TxRx) for 129Xe (34.66 MHz)

"X coil", 125 MHz (Rx)



Tunable CSA hyperthermia module (97 MHz, Tx)

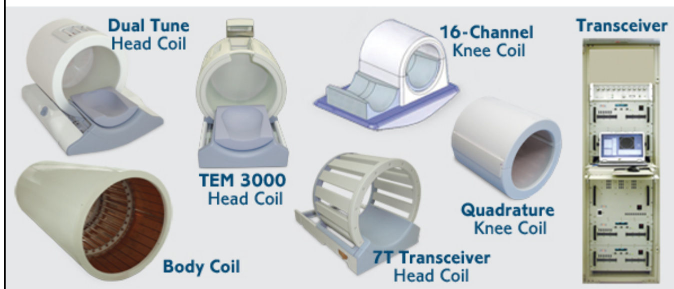
4-channel CSA array (TxRx)

125-MHz CSA

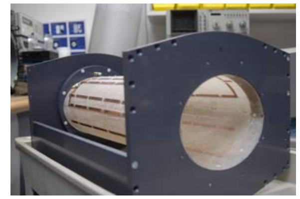
Lung coil (TxRx) with differential transmitter for 129Xe (34.66 MHz)

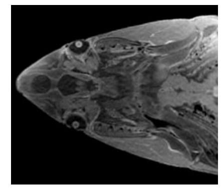
2-channel strip-line array (TxRx)

http://www.berlin.ptb.de/en/org/8/81/811/ResearchTopics/CoilDevelopment.html



http://www.mrinstruments.com/





Images from www.dfl.org

Rotating Frame Bloch Equation

$$\frac{d\mathbf{M}_{rot}}{dt} = \mathbf{M}_{rot} \times \gamma \mathbf{B}_{eff}$$

$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}; \quad \omega_{rot} = \begin{bmatrix} 0 \\ 0 \\ -\omega \end{bmatrix}$$

Note: we use the RF frequency to define the rotating frame. If this RF frequency is on-resonance, then the main B0 field doesn't cause any precession in the rotating frame. However, if the RF frequency is off-resonance, then there will be a net precession in the rotating frame that is give by the difference between the RF frequency and the local Larmor frequency.

$$Let\ \mathbf{B}_{rot} = B_1(t)\mathbf{i} + B_0\mathbf{k}$$

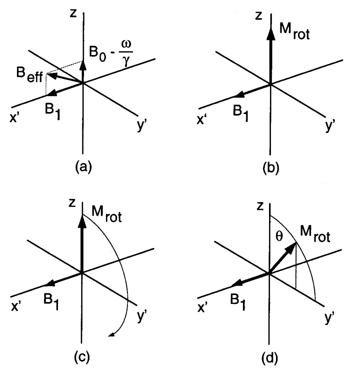
$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}$$

$$= B_1(t)\mathbf{i} + \left(B_0 - \frac{\omega}{\gamma}\right)\mathbf{k}$$

$$\text{If } \omega = \omega_0$$

$$= \gamma B_0$$

$$\text{Then } \mathbf{B}_{eff} = B_1(t)\mathbf{i}$$



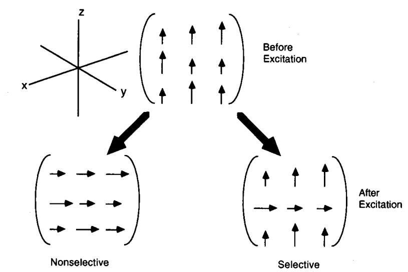
Flip angle

$$\theta = \int_0^{\tau} \omega_1(s)ds$$

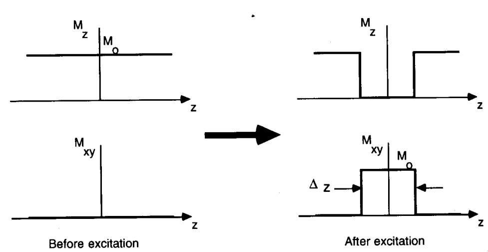
where

$$\omega_1(t) = \gamma B_1(t)$$

Nishimura 1996

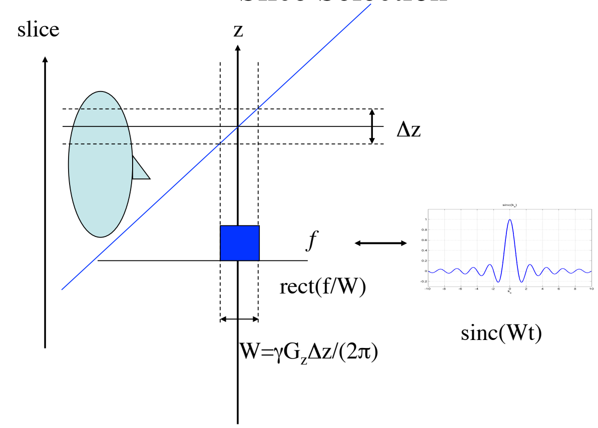


Nishimura 1996



Nishimura 1996

Slice Selection



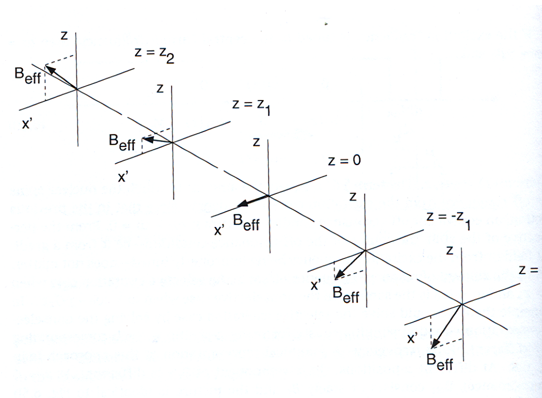
$$Let\ \mathbf{B}_{rot} = B_1(t)\mathbf{i} + \left(B_0 + \gamma G_z z\right)\mathbf{k}$$

$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}$$

$$= B_1(t)\mathbf{i} + \left(B_0 + \gamma G_z z - \frac{\omega}{\gamma}\right)\mathbf{k}$$

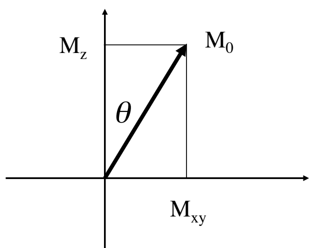
If $\omega = \omega_0$

$$\mathbf{B}_{eff} = B_1(t)\mathbf{i} + \left(\gamma G_z z\right)\mathbf{k}$$



Nishimura 1996

Small Tip Angle Approximation



For small θ

$$M_z = M_0 \cos\theta \approx M_0$$

$$M_{xy} = M_0 \sin\theta \approx M_0\theta$$

Excitation k-space

At each time increment of width $\Delta\tau$, the excitation $B_1(\tau)$ produces an increment in magnetization of the form $\Delta M_{xy} \approx jM_0\gamma B_1(\tau)\Delta\tau$ (small tip angle approximation)

In the presence of a gradient, this will accumulate phase of the form $\varphi = -\gamma\int_\tau^t zG_z(s)ds$, such that the incremental magnetization at time t is

$$\Delta M_{xy}(t,z\,;\,\tau) = jM_0\gamma B_1(\tau)\exp\left(-j\gamma\int_\tau^t zG_z(s)ds\right)\Delta\tau$$

Integrating over all time increments, we obtain

$$M_{xy}(t,z) = jM_0\int_{-\infty}^t \gamma B_1(\tau)\exp\left(-j\gamma\int_\tau^t zG_z(s)ds\right)d\tau$$

$$= jM_0\int_{-\infty}^t \gamma B_1(\tau)\exp\left(j2\pi k(\tau,t)z\right)d\tau$$

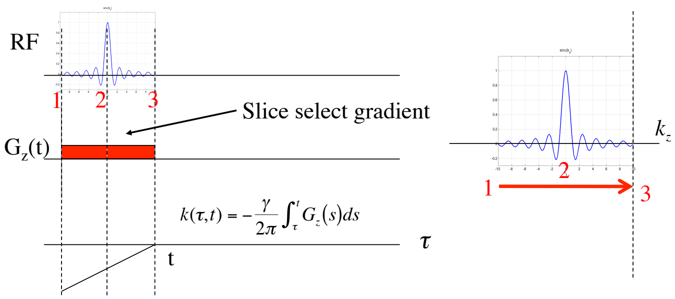
where $k(\tau,t) = -\frac{\gamma}{2\pi}\int_\tau^t G_z(s)ds$

Pauly et al 1989

Excitation k-space

$$M_{xy}(t,z) = jM_0 \int_{-\infty}^{t} \gamma B_1(\tau) \exp\left(j2\pi k(\tau,t) z\right) d\tau$$

This has the form of an inverse Fourier transform, where we are integrating the contributions of the field $B_1(\tau)$ at the k-space point $k(\tau,t)$.



Excitation k-space

For a constant gradient:

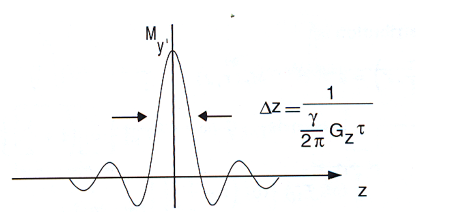
$$k_z(\tau,t) = \frac{\gamma}{2\pi} G_z(\tau - t) \Rightarrow \tau = \frac{2\pi}{\gamma G_z} k_z + t, \quad d\tau = \frac{2\pi}{\gamma G_z} dk_z$$

$$\begin{aligned} M_{xy}(t,z) &= jM_0 \int_{-\infty}^{t} \gamma B_1(\tau) \exp\left(j2\pi k(\tau,t) z\right) d\tau \\ &= jM_0 \int_{-\infty}^{t} \gamma B_1\left(\frac{2\pi}{\gamma G_z} k_z + t\right) \exp\left(j2\pi k_z z\right) \frac{2\pi}{\gamma G_z} dk_z \\ &= jM_0 \exp\left(-j\gamma G_z t z\right) \int_{-\infty}^{t} \gamma B_1(k_z) \exp\left(j2\pi k_z \left(\frac{\gamma G_z}{2\pi} z\right)\right) dk_z \\ &= jM_0 \exp\left(-j\omega(z) t\right) F^{-1}\left[\gamma B_1(k_z)\right]_{\frac{\gamma G_z}{2\pi} z} \end{aligned}$$

Small Tip Angle Example

$$B_1(t) = B_1 rect\left(\frac{t}{\tau}\right)$$

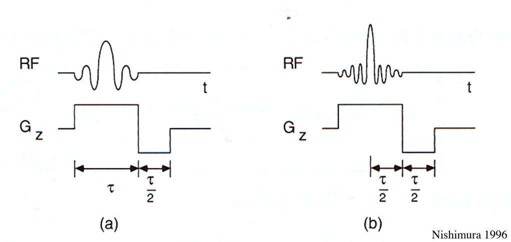
$$\begin{aligned} M_r(\tau/2, z) &= jM_0 \exp(-j\omega(z)\tau/2) F^{-1}\left(\omega_1 rect\left(\frac{k_z}{\tau}\right)\right)\Big|_{\frac{\gamma G_z}{2\pi} z} \\ &= jM_0 \exp(-j\omega(z)\tau/2) \omega_1 \tau \operatorname{sinc}\left(\frac{\gamma G_z \tau}{2\pi} z\right) \end{aligned}$$



Nishimura 1996

Refocusing

$$\begin{aligned} M_r(\tau,z) &= \exp\left(j\omega(z)\tau/2\right) M_r(\tau,z) \\ &= jM_0 \exp\left(j\omega(z)\tau/2\right) \exp\left(-j\omega(z)\tau/2\right) F^{-1}\left[\gamma B_1(k_z)\right]_{\frac{\gamma G_z}{2\pi} z} \\ &= jM_0 F^{-1}\left[\gamma B_1(k_z)\right]_{\frac{\gamma G_z}{2\pi} z} \end{aligned}$$

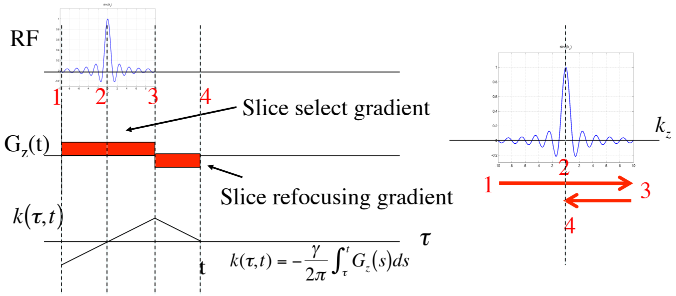


Nishimura 1996

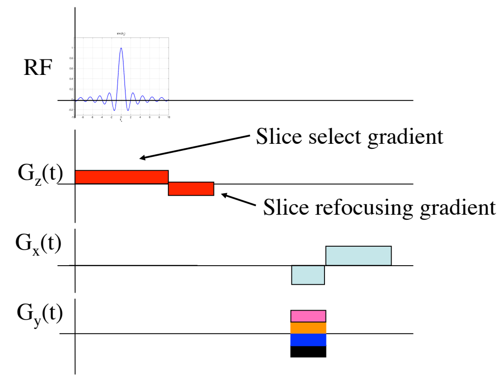
Refocusing

$$M_{xy}(t,z) = jM_0 \int_{-\infty}^{t} \gamma B_1(\tau) \exp\big(j2\pi k(\tau,t) z\big) d\tau$$

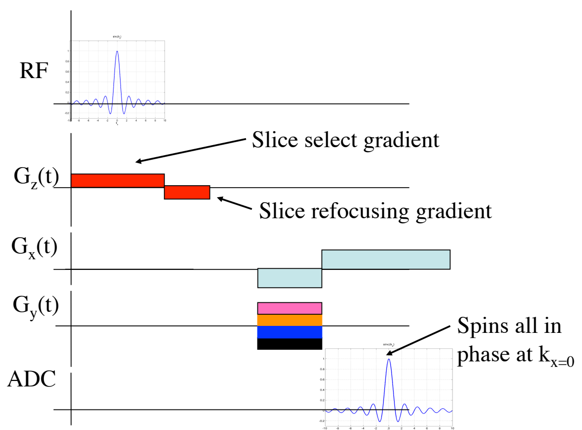
This has the form of an inverse Fourier transform, where we are integrating the contributions of the field $B_1(\tau)$ at the k-space point $k(\tau,t)$.



Slice Selection



Gradient Echo

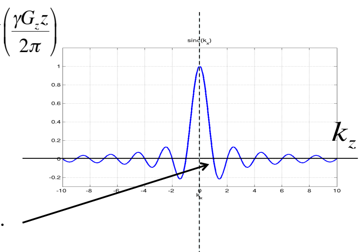


Small Tip Angle Example

$$B_1(t) = A\,\mathrm{sinc}(t/\tau)\left(0.5 + 0.46\cos\left(\frac{2\pi t}{\tau}\right)\right)$$

$$= A\,\mathrm{sinc}(t/\tau)w(t)$$

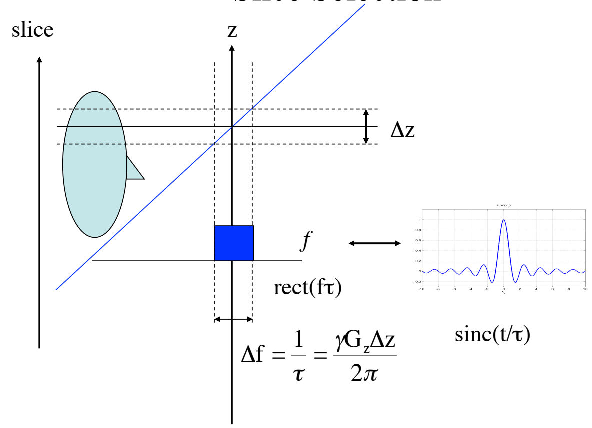
$$F^{-1}(B_1(k_z))\Big|_{\frac{\gamma G_z z}{2\pi}} = A\tau\ rect\left(\frac{\gamma G_z z\tau}{2\pi}\right) * W\left(\frac{\gamma G_z z}{2\pi}\right)$$



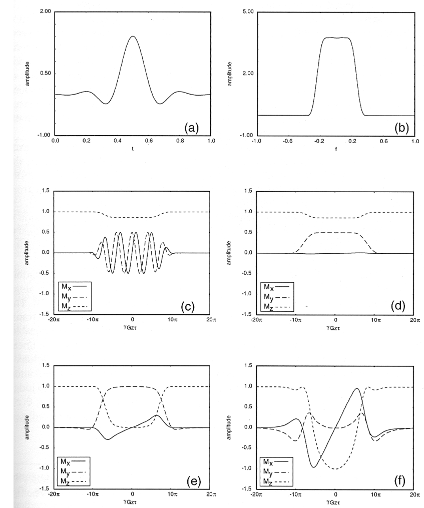
First zero in k_z space is at $\frac{\gamma G_z \tau}{2\pi}$.

Therefore, width of the rect function is $\Delta z = \frac{2\pi}{\gamma G_z \tau}$

Slice Selection



$$\Delta f = \frac{1}{\tau} = \frac{\gamma G_z \Delta z}{2\pi}$$



Nishimura 1996

Example

$\Delta z = 5\ mm; \tau = 400\ \mu\text{sec};\ \theta = \pi/2$

$$G_z = \frac{2\pi}{\gamma \Delta z \tau} = \frac{1}{(4257 Hz/G)(0.5cm)(400e-6)} = 1.175\ G/cm$$

$$\theta \approx \gamma \int_0^T B_1 \operatorname{sinc}\left(\frac{s - T/2}{\tau}\right) ds \approx \gamma B_1 \cdot (\text{area of sinc}) = \gamma B_1 \tau$$

$$B_1 = \frac{\theta}{\gamma \tau} = \frac{\pi/2}{2\pi(4257 Hz/G)(400e-6)} = 0.1468\ G$$

Time-Bandwidth Product (TBW)

$$\operatorname{sinc}(t/\tau)\, rect\left(\frac{t}{2N\tau}\right) \Rightarrow \tau\, rect(\tau f) * 2N\tau \operatorname{sinc}(2N\tau f)$$

N = number of zeros in Sinc

Duration $= 2N\tau$

Bandwidth $= \frac{1}{\tau} \quad \Rightarrow \Delta z = \frac{2\pi}{\gamma G_z \tau}$

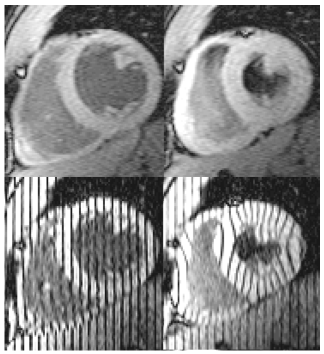
Transition Width $\approx \frac{1}{2N\tau} \quad \Rightarrow \Delta z' = \frac{2\pi}{\gamma G_z 2N\tau}$

Time – Bandwidth Product (TBW) $= 2N\tau \frac{1}{\tau} = 2N$

also, TBW $= \frac{\text{Bandwidth}}{\text{Transition Width}}$

For a fixed duration pulse, we can increase TBW by increasing the Bandwidth.
(Note: this will also lead to an increase in N).
This will require a higher B1 amplitude and a higher gradient to keep the slice width constant -- note that with higher TBW the physical transition width then decreases.

Cardiac Tagging



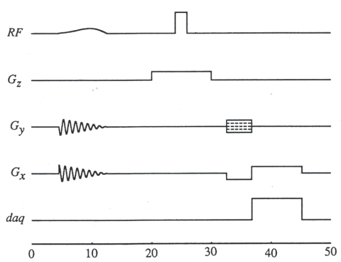
Multi-dimensional Excitation k-space

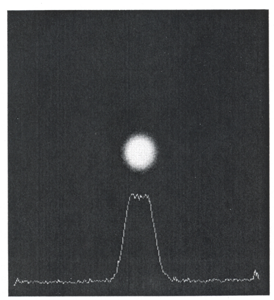
$$M_{xy}(t,\mathbf{r}) = jM_0 \int_{-\infty}^{t} \omega_1(\tau) \exp\left(-j\gamma \int_{\tau}^{t} \mathbf{G}(s) \cdot \mathbf{r}\, ds\right) d\tau$$
$$= jM_0 \int_{-\infty}^{t} \omega_1(\tau) \exp\left(j2\pi \mathbf{k}(\tau) \cdot \mathbf{r}\right) d\tau$$

where $\mathbf{k}(\tau) = -\frac{\gamma}{2\pi} \int_{\tau}^{t} \mathbf{G}(t')\, dt'$

Pauly et al 1989

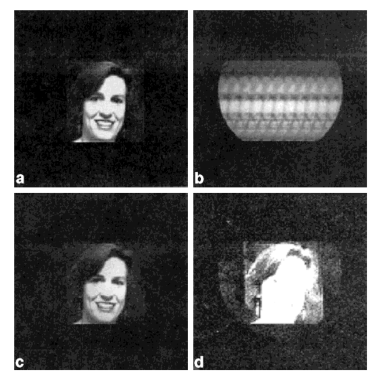
Excitation k-space





Pauly et al 1989

Excitation k-space



Panych MRM 1999