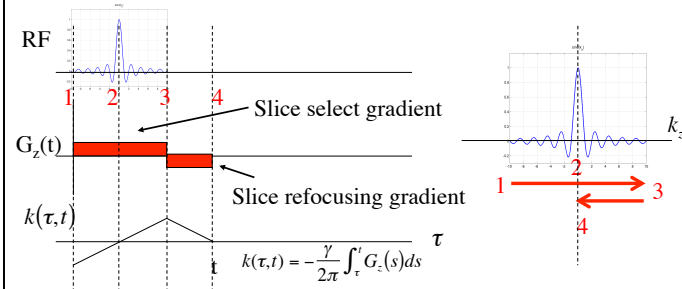Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2009
MRI Lecture 5

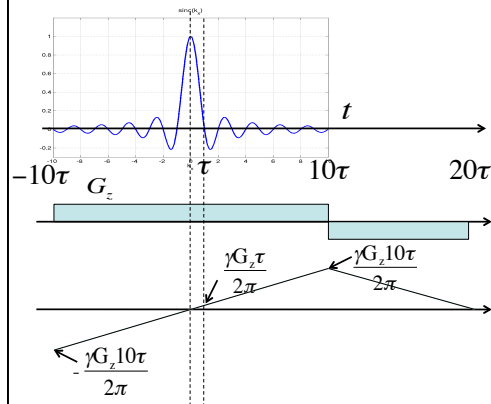## Refocusing

$$M_{xy}(t,z) = jM_0 \int_{-\infty}^{t} \gamma B_1(\tau)\exp\left(j2\pi k(\tau,t)z\right)d\tau$$

This has the form of an inverse Fourier transform, where we are
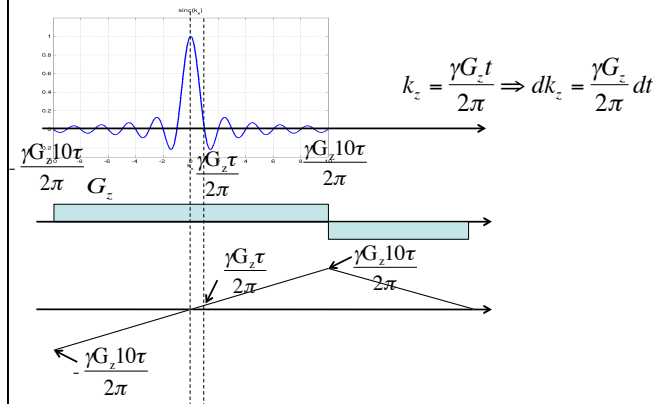integrating the contributions of the field $B_1(\tau)$ at the k - space point $k(\tau,t)$.

RF

1 2 3 4   ← Slice select gradient

$G_z(t)$

← Slice refocusing gradient

$k(\tau,t)$

$k(\tau,t) = -\dfrac{\gamma}{2\pi}\int_{\tau}^{t}G_z(s)ds$

k_z

Small Tip Angle Example

$$B_1(t) = A\sin c(t/\tau)\, rect\left(\frac{t}{20\tau}\right)$$

$sinc(k_z)$

-10τ τ 10τ 20τ

G_z

$\dfrac{\gamma G_z\tau}{2\pi}$ $\dfrac{\gamma G_z 10\tau}{2\pi}$

$-\dfrac{\gamma G_z 10\tau}{2\pi}$

Small Tip Angle Example

$$B_1(k_z) = A\sin c\left(\frac{k_z 2\pi}{\gamma G_z\tau}\right) rect\left(\frac{k_z 2\pi}{20\gamma G_z\tau}\right)$$

$sinc(k_z)$

$$k_z = \frac{\gamma G_z t}{2\pi} \Rightarrow dk_z = \frac{\gamma G_z}{2\pi}dt$$

$-\dfrac{\gamma G_z 10\tau}{2\pi}$ $\dfrac{\gamma G_z\tau}{2\pi}$ $\dfrac{\gamma G_z 10\tau}{2\pi}$

G_z

$\dfrac{\gamma G_z\tau}{2\pi}$ $\dfrac{\gamma G_z 10\tau}{2\pi}$

$-\dfrac{\gamma G_z 10\tau}{2\pi}$

Small Tip Angle Example

$$B_1(k_z) = A \operatorname{sinc}\left(\frac{k_z 2\pi}{\gamma G_z \tau}\right) rect\left(\frac{k_z 2\pi}{20\gamma G_z \tau}\right)$$

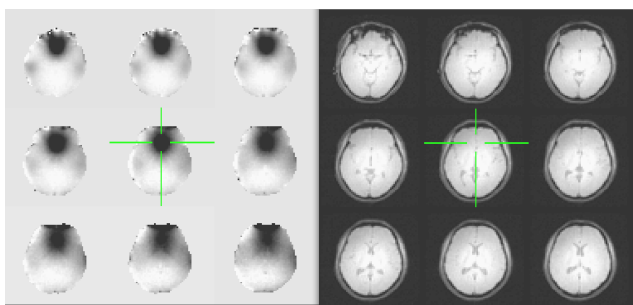$$M_{xy}(t,z) = jM_0 \int_{-\infty}^{t} \gamma B_1(\tau) \exp\left(j2\pi k(\tau,t)z\right) d\tau$$

$$= jM_0 \int_{-\infty}^{\infty} \gamma A \operatorname{sinc}\left(\frac{k_z 2\pi}{\gamma G_z \tau}\right) rect\left(\frac{k_z 2\pi}{20\gamma G_z \tau}\right) \exp\left(j2\pi k_z z\right)\frac{2\pi}{\gamma G_z} dk_z$$

$$= jM_0 \gamma A \frac{2\pi}{\gamma G_z} F^{-1}\left(\operatorname{sinc}\left(\frac{k_z 2\pi}{\gamma G_z \tau}\right) rect\left(\frac{k_z 2\pi}{20\gamma G_z \tau}\right)\right)$$

$$= jM_0 \gamma A \tau \cdot rect\left(\frac{\gamma G_z \tau}{2\pi} z\right) * \frac{20\gamma G_z \tau}{2\pi}\operatorname{sinc}\left(\frac{20\gamma G_z \tau}{2\pi} z\right)$$
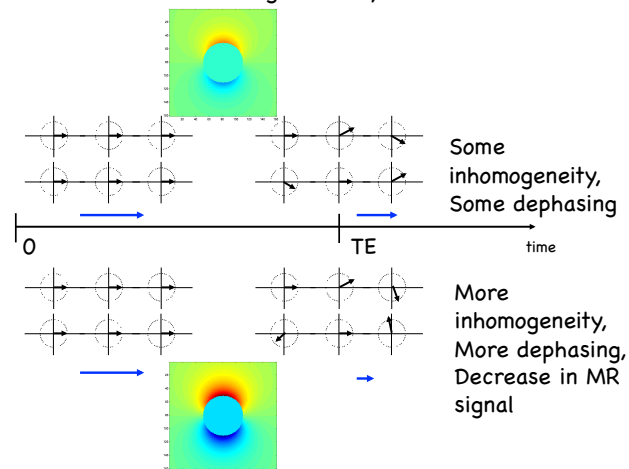
Static Inhomogeneities

In the ideal situation, the static magnetic field is totally uniform and the reconstructed object is determined solely by the applied gradient fields. In reality, the magnet is not perfect and will not be totally uniform. Part of this can be addressed by additional coils called "shim" coils, and the process of making the field more uniform is called "shimming". In the old days this was done manually, but modern magnets can do this automatically.

In addition to magnet imperfections, most biological samples are inhomogeneous and this will lead to inhomogeneity in the field. This is because, each tissue has different magnetic properties and will distort the field.

Field Inhomogeneities

Signal Decay

Some inhomogeneity, Some dephasing

0   TE   time

More inhomogeneity, More dephasing, Decrease in MR signal

Static Inhomogeneities

The spatial nonuniformity in the field can be modeled by adding an additional term to our signal equation.

$$s_r(t) = \int_V M(\vec{r}, t)dV$$

$$= \int_x \int_y \int_z M(x, y, z, 0) e^{-t/T_2(\vec{r})} e^{-j\omega_0 t} e^{-j\omega_E(\vec{r})t} \exp\left(-j\gamma \int_o^t \vec{G}(\tau) \cdot \vec{r} d\tau\right) dxdydz$$

The effect of this nonuniformity is to cause the spins to dephase with time and thus for the signal to decrease more rapidly. To first order this can be modeled as an additional decay term of the form

$$s_r(t) = \int_x \int_y \int_z M(x, y, z, 0) e^{-t/T_2(\vec{r})} e^{-t/T_2'(\vec{r})} e^{-j\omega_0 t} \exp\left(-j\gamma \int_o^t \vec{G}(\tau) \cdot \vec{r} d\tau\right) dxdydz$$

T_2^* decay

The overall decay has the form.

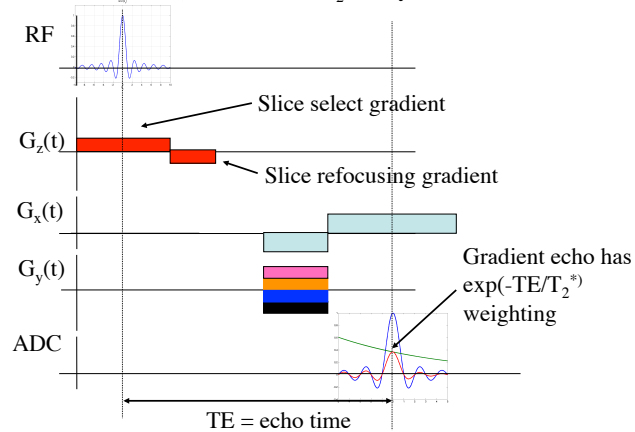$$\exp\left(-t/T_2^*(\vec{r})\right)$$

where

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$

Due to random motions of spins. Not reversible.

Due to static inhomogeneities. Reversible with a spin-echo sequence.
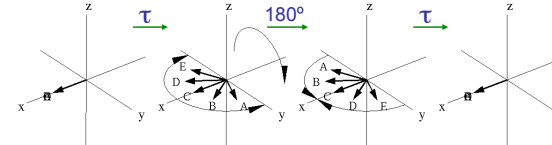

T_2^* decay

Gradient echo sequences exhibit T_2^* decay.

RF

$G_z(t)$ — Slice select gradient

— Slice refocusing gradient

$G_x(t)$

$G_y(t)$

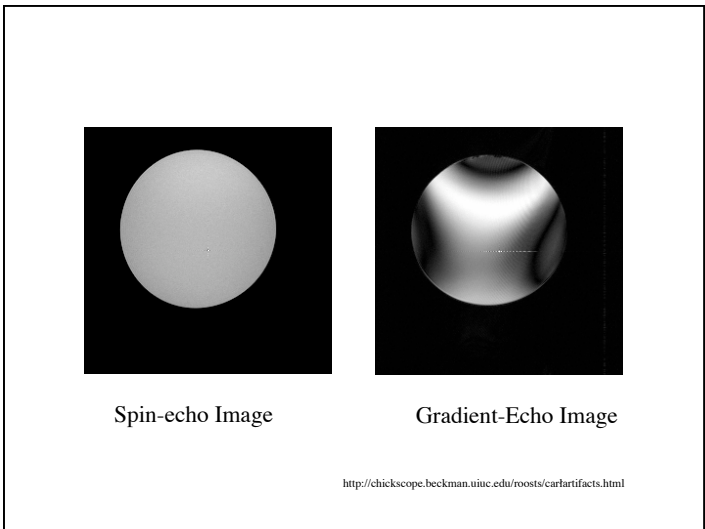Gradient echo has $\exp(-TE/T_2^*)$ weighting

ADC
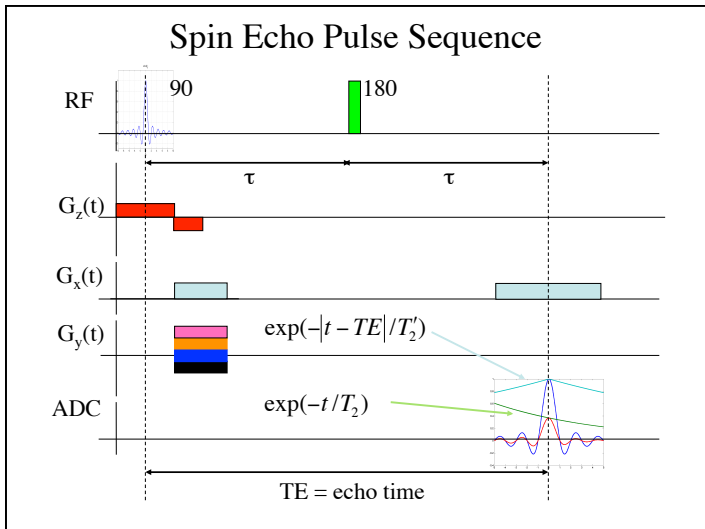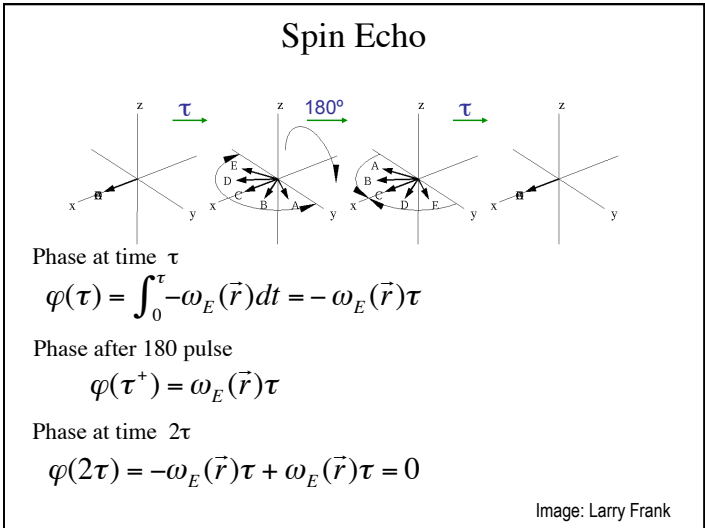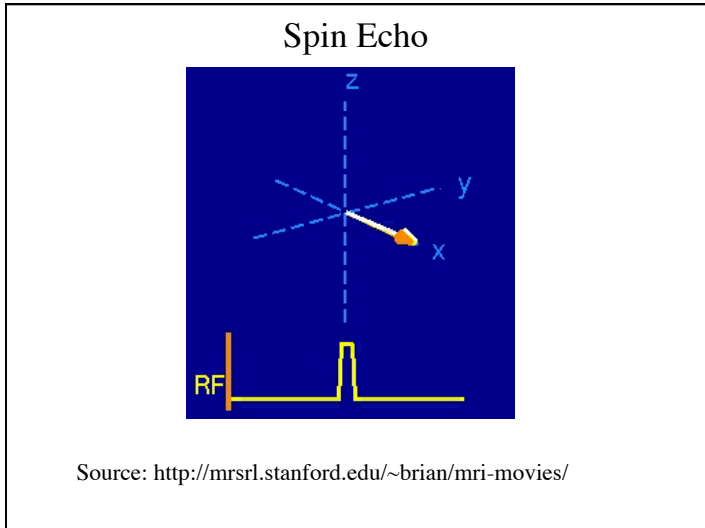
TE = echo time

Spin Echo

Discovered by Erwin Hahn in 1950.

The spin-echo can refocus the dephasing of spins due to static inhomogeneities. However, there will still be T_2 dephasing due to random motion of spins.

There is nothing that nuclear spins will not do for you, as long as you treat them as human beings. Erwin Hahn

Image: Larry Frank

Spin Echo

Source: http://mrsrl.stanford.edu/~brian/mri-movies/

Spin Echo

Phase at time τ

$$\varphi(\tau) = \int_0^\tau -\omega_E(\vec{r})dt = -\omega_E(\vec{r})\tau$$

Phase after 180 pulse

$$\varphi(\tau^+) = \omega_E(\vec{r})\tau$$

Phase at time 2τ

$$\varphi(2\tau) = -\omega_E(\vec{r})\tau + \omega_E(\vec{r})\tau = 0$$

Image: Larry Frank

Spin Echo Pulse Sequence

$\exp(-|t - TE|/T_2')$

$\exp(-t/T_2)$

TE = echo time

Spin-echo Image   Gradient-Echo Image

http://chickscope.beckman.uiuc.edu/roosts/carl/artifacts.html

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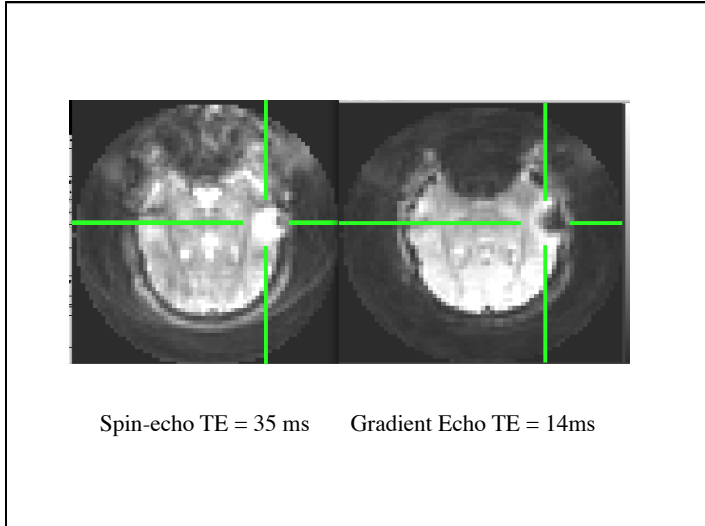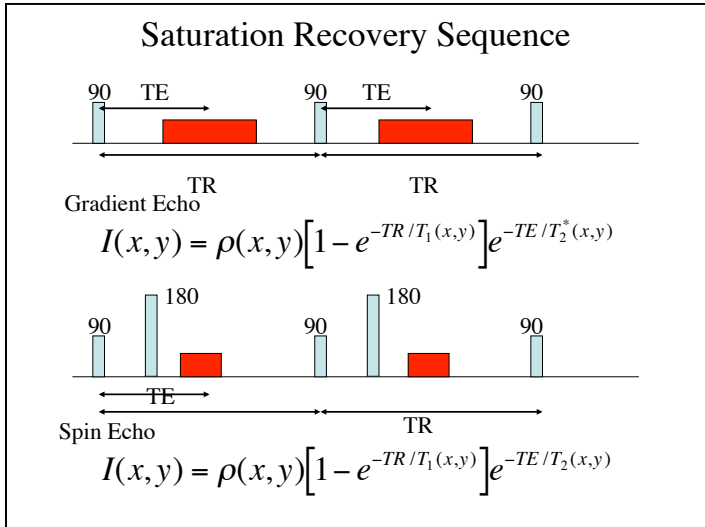Spin-echo TE = 35 ms   Gradient Echo TE = 14ms

Image Contrast

Different tissues exhibit different relaxation rates, T_1, T_2, and T_2^*. In addition different tissues can have different densities of protons. By adjusting the pulse sequence, we can create contrast between the tissues. The most basic way of creating contrast is adjusting the two sequence parameters: TE (echo time) and TR (repetition time).

Saturation Recovery Sequence

Gradient Echo

$$I(x,y) = \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right]e^{-TE/T_2^*(x,y)}$$

Spin Echo

$$I(x,y) = \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right]e^{-TE/T_2(x,y)}$$

T1-Weighted Scans

Make TE very short compared to either T_2 or T_2^*. The resultant image has both proton and T_1 weighting.

$$I(x,y) \approx \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right]$$

T2-Weighted Scans

Make TR very long compared to T_1 and use a spin-echo pulse sequence. The resultant image has both proton and T_2 weighting.
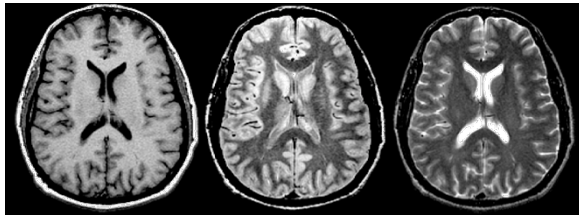
$$I(x,y) \approx \rho(x,y)e^{-TE/T_2}$$

Proton Density Weighted Scans

Make TR very long compared to T_1 and use a very short TE. The resultant image is proton density weighted.
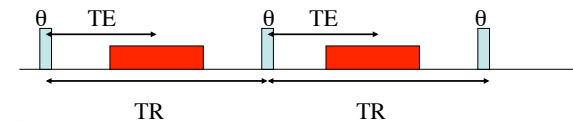
$$I(x,y) \approx \rho(x,y)$$

Example

T$_1$-weighted   Density-weighted   T$_2$-weighted

Tissue	Proton Density	T1 (ms)	T2 (ms)
Csf	1.0	4000	2000
Gray	0.85	1350	110
White	0.7	850	80

FLASH sequence

Gradient Echo

$$I(x,y) = \rho(x,y)\frac{\left[1-e^{-TR/T_1(x,y)}\right]\sin\theta}{\left[1-e^{-TR/T_1(x,y)}\cos\theta\right]}\exp(-TE/T_2^*)$$

Signal intensity is maximized at the Ernst Angle

$$\theta_E = \cos^{-1}\left(\exp(-TR/T_1)\right)$$

FLASH equation assumes no coherence from shot to shot. In practice this is achieved with RF spoiling.

FLASH sequence

Flash signal intensity; T1 = 1000 ms

$$\theta_E = \cos^{-1}\left(\exp(-TR/T_1)\right)$$

Inversion Recovery

TI   TE

TR

$$I(x,y) = \rho(x,y)\left[1 - 2e^{-TI/T_1(x,y)} + e^{-TR/T_1(x,y)}\right]e^{-TE/T_2(x,y)}$$

Intensity is zero when inversion time is

$$TI = -T_1 \ln\left[\frac{1 + \exp(-TR/T_1)}{2}\right]$$

Inversion Recovery

GE Medical Systems 2003