BENG280A, Principles of Biomedical Imaging 10/31/12
Fall Quarter 2012

HOMEWORK #5
Due at 5 pm on Wednesday 11/7/12

Homework policy: Homeworks can be turned in during class or to the TA's mailbox in the Graduate Student Lounge. Late homeworks will be marked down by 20% per day. If you know that you need to turn in a homework late because of an emergency or academic travel, please let the TA know ahead of time. Collaboration is encouraged on homework assignments, however, the homework that you submit should reflect your own understanding of the material.

Readings: Section 6 in the textbook; Chapters 1 and 2.1-2.3 in the EEG printed material handed out in class.

Problem 1
You are asked to design an RF and gradient pulse sequence that achieves the following profile: $M_{xy}(z) = \frac{1}{4}\left(1+\cos(4\pi z)\right)^2$. The maximum available B_1 radiofrequency field is 0.2 Gauss and the maximum available gradient is 4 G/cm.

a) First compute the Fourier transform of the pulse profile. You may find it useful to use the convolution/multiplication theorem.
b) Next use the small tip angle approximation to determine the desired flip angles for each pulse. Compute the temporal width of each pulse required to achieve the desired flip angles, assuming that each pulse uses the maximum available B_1 field.
c) Derive a plot of the desired k_z versus time. Use this plot to design your gradient waveforms. Make sure to clearly label all amplitudes and timing parameters.
d) Draw a quiver diagram that shows the orientation of the spins from each RF pulse at the end of the pulse sequence. Discuss how the sum of the spin profiles achieves the desired overall profile. In specific, test out the locations where you expect the profile to equal 0, ½ or 1. Make sure to take into account the flip angles of the RF pulses that you used.

Problem 2

(a) Define the fMRI signals during baseline and activation as $s_B = M_0 \exp\left(-R_{2,B}^* TE\right)$ and $s_A = M_0 \exp\left(-R_{2,A}^* TE\right)$, respectively. Use a Taylor series approximation to show that the relative fMRI signal change can be approximated as $\frac{s_A - s_B}{s_B} \approx -TE\left(R_{2,A}^* - R_{2,B}^*\right)$. Under what circumstances is this approximation valid? Based on this finding, your advisor tells you to acquire data at a really long echo time, since that will maximize the relative signal change. What is wrong with this suggestion?

(b) Next consider the signal difference $s_A - s_B$. Based on the approximation above, show that this can be written as $s_A - s_B \approx -TE\left(R_{2,A}^* - R_{2,B}^*\right) M_0 \exp(-R_{2,B}^* TE)$. For what value of TE is this expression maximized? Verify your answer by plotting the signal difference as a function of TE for $R_{2,B}^* = 25\ \text{s}^{-1}$ and $R_{2,A}^* = 22\ \text{s}^{-1}$.

(c) Based upon your findings from parts (a) and (b), what value of TE would you recommend using? Provide an explanation of your answer.

Problem 3 (OPTIONAL Bonus Exercise)
Consider the RF pulse design example from MRI Lecture 6 where we had a desired excitation pattern $M_{xy}(x) = M_0 \cos(4\pi x)$.

(a) In class, we used the small tip angle approximation to determine the desired flip angles for each pulse. Assume that the maximum RF amplitude available is 0.3 G. What is the temporal width of each pulse (i.e. what is required to achieve the desired flip angle)?
(b) If these finite-width pulses are played out during the gradient, the effect can be modeled by replacing each Dirac delta function with a shifted rect function of width K. Determine the value of K (make use of the gradients and timing information we derived in class and the width of the RF pulses).
(c) What is the effect of the finite-pulse width on the desired excitation pattern? (note: instead of the Fourier transform of two shifted Dirac delta functions, you will be considering the Fourier transform of two shifted rect functions).
(d) What design change could you make to avoid this effect? (i.e. can you think of a simple solution in which k-space doesn't change while you are playing out the finite-width RF-pulse).

MATLAB PROBLEM

1. Look up the definition of correlation coefficient (e.g. on Wikipedia) and write a program to compute the correlation coefficient between two vectors **X** and **Y** (give your function a meaningful name but do NOT name it corrcoef). Your program should also return the p-value associate with the correlation coefficient (you will want to look up how to compute the p-value). Compare your results to MATLAB's corrcoef function. You should get the same result.
2. Use your function to compute correlation maps for the fMRI dataset from the sample brain (assign1.mat on webpage). Note that this dataset is stored in the matrix **rawdata_r**, and has dimensions of 64x64x6x164. This is a 4D volume where the first 3 dimensions are the x,y, and z coordinates, and the last dimension index time. Use the reference vector **ref** from the *.mat file. Can you detect functional activation in the brain? Where in the brain is it? What happens when you apply a threshold to your maps? What p-value appears to give the best maps? Repeat this with the reference vector **ref0**.
3. Use the MATLAB etime function to see much time it takes to compute the correlation maps. Remember that using **for** loops in MATLAB is really inefficient. Can you come up with a faster way of computing the correlation coefficients? Hint: You will want to make judicious use of matrix multiplication. Some useful commands are: *reshape* and *bsxfun*