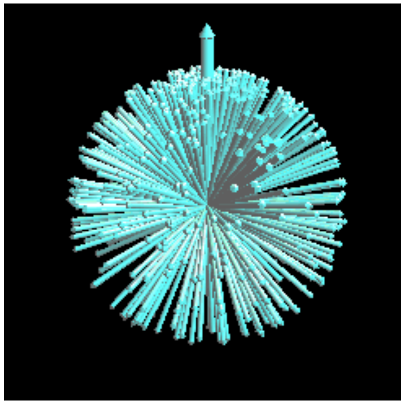
Bioengineering 280A
Principles of Biomedical Imaging

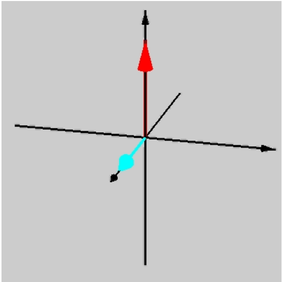
Fall Quarter 2012
MRI Lecture 6

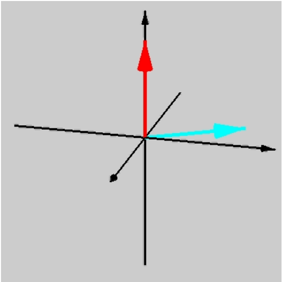
RF Excitation



http://www.drcmr.dk/main/content/view/213/74/

RF Excitation

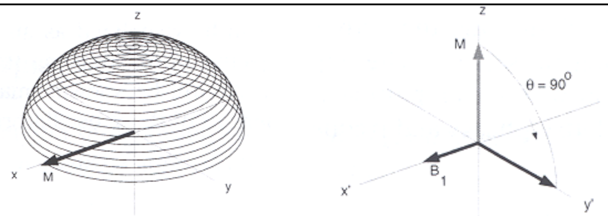




http://www.eecs.umich.edu/%7EdnollBME516/

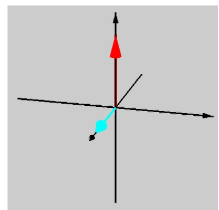
Rotating Frame of Reference

Reference everything to the magnetic field at isocenter.



a) Laboratory frame behavior of **M** b) Rotating frame behavior of **M**

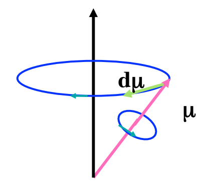
Images & caption: Nishimura, Fig. 3.3



http://www.eecs.umich.edu/%7Ednoll/BME516/

Precession

$$\frac{d\mu}{dt} = \mu \times \gamma \mathbf{B}$$

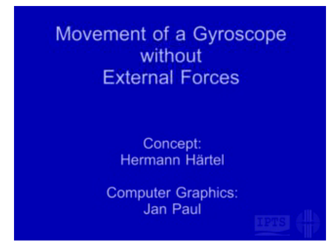


Analogous to motion of a gyroscope

Precesses at an angular frequency of

$\omega = \gamma B$

This is known as the **Larmor** frequency.



http://www.astrophysik.uni-kiel.de/~hhaertel/mpg_e/gyros_free.mpg

Rotating Frame Bloch Equation

$$\frac{d\mathbf{M}_{rot}}{dt} = \mathbf{M}_{rot} \times \gamma \mathbf{B}_{eff}$$

$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}; \quad \omega_{rot} = \begin{bmatrix} 0 \\ 0 \\ -\omega \end{bmatrix}$$

Note: we use the RF frequency to define the rotating frame. If this RF frequency is on-resonance, then the main B0 field doesn't cause any precession in the rotating frame. However, if the RF frequency is off-resonance, then there will be a net precession in the rotating frame that is give by the difference between the RF frequency and the local Larmor frequency.

$$Let \ \mathbf{B}_{rot} = B_1(t)\mathbf{i} + B_0\mathbf{k}$$

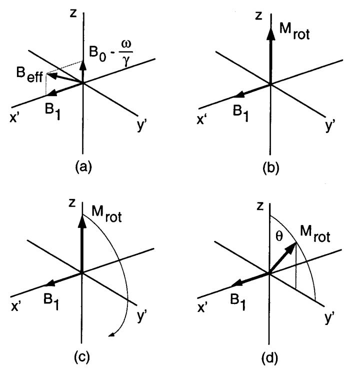
$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}$$

$$= B_1(t)\mathbf{i} + \left(B_0 - \frac{\omega}{\gamma}\right)\mathbf{k}$$

$$\text{If } \omega = \omega_0$$

$$= \gamma B_0$$

$$\text{Then } \mathbf{B}_{eff} = B_1(t)\mathbf{i}$$



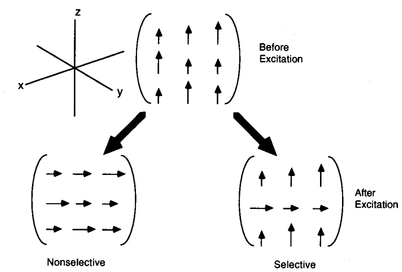
Flip angle

$$\theta = \int_0^{\tau} \omega_1(s)\,ds$$

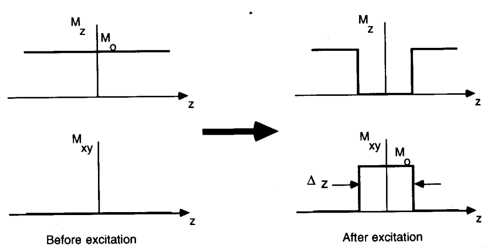
where

$$\omega_1(t) = \gamma B_1(t)$$

Nishimura 1996



Nishimura 1996



Nishimura 1996

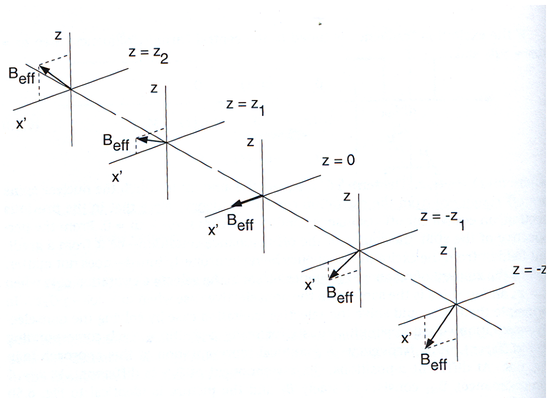
Let $\mathbf{B}_{rot} = B_1(t)\mathbf{i} + \left(B_0 + \gamma G_z z\right)\mathbf{k}$

$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}$$

$$= B_1(t)\mathbf{i} + \left(B_0 + \gamma G_z z - \frac{\omega}{\gamma}\right)\mathbf{k}$$

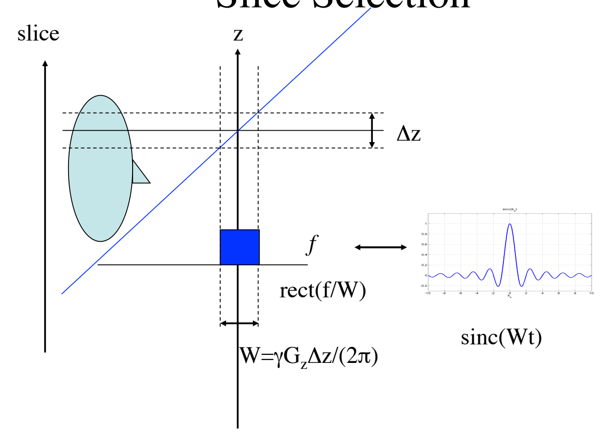
If $\omega = \omega_0$

$$\mathbf{B}_{eff} = B_1(t)\mathbf{i} + \left(\gamma G_z z\right)\mathbf{k}$$

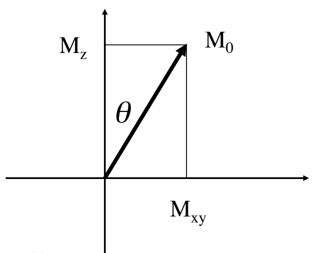


Nishimura 1996

Slice Selection



Small Tip Angle Approximation

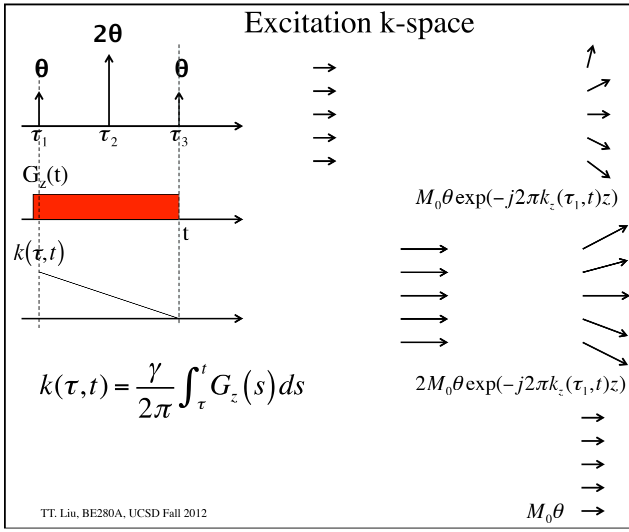


For small θ

$$M_z = M_0 \cos\theta \approx M_0$$

$$M_{xy} = M_0 \sin\theta \approx M_0\theta$$

Excitation k-space



$$k(\tau,t) = \frac{\gamma}{2\pi}\int_\tau^t G_z(s)\,ds$$

Excitation k-space

At each time increment of width $\Delta\tau$, the excitation $B_1(\tau)$ produces an increment in magnetization of the form $\Delta M_{xy} \approx jM_0\gamma B_1(\tau)\Delta\tau$ (small tip angle approximation)

In the presence of a gradient, this will accumulate phase of the form $\varphi = -\gamma\int_\tau^t zG_z(s)\,ds$, such that the incremental magnetization at time t is

$$\Delta M_{xy}(t,z\,;\,\tau) = jM_0\gamma B_1(\tau)\exp\left(-j\gamma\int_\tau^t zG_z(s)\,ds\right)\Delta\tau$$

Integrating over all time increments, we obtain

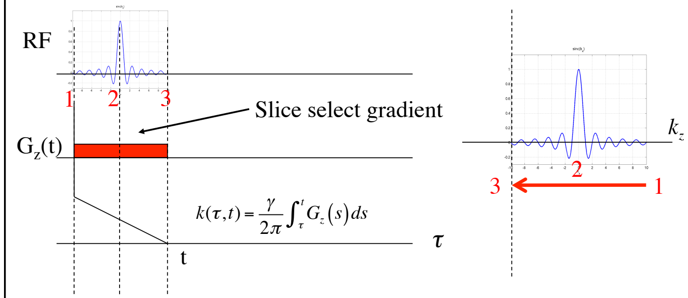
$$M_{xy}(t,z) = jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp\left(-j\gamma\int_\tau^t zG_z(s)\,ds\right)d\tau$$
$$= jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp\left(-j2\pi k(\tau,t)z\right)d\tau$$

where $k(\tau,t) = \frac{\gamma}{2\pi}\int_\tau^t G_z(s)\,ds$

Excitation k-space

$$M_{xy}(t,z) = jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp\left(-j2\pi k(\tau,t)z\right)d\tau$$

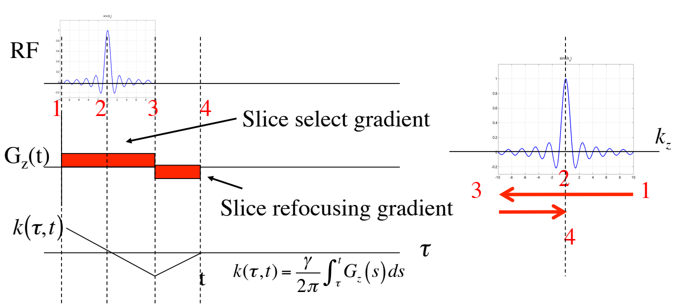
This has the form of an inverse Fourier transform, where we are integrating the contributions of the field $B_1(\tau)$ at the k - space point $k(\tau,t)$.



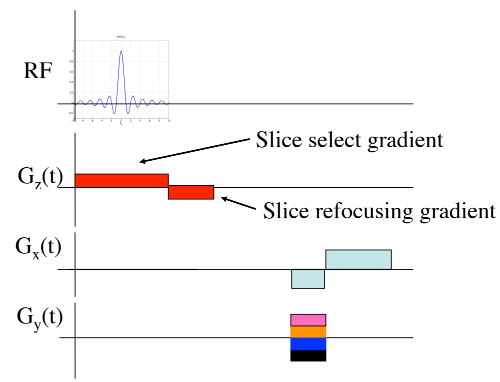
Refocusing

$$M_{xy}(t,z) = jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp\left(-j2\pi k(\tau,t)z\right)d\tau$$

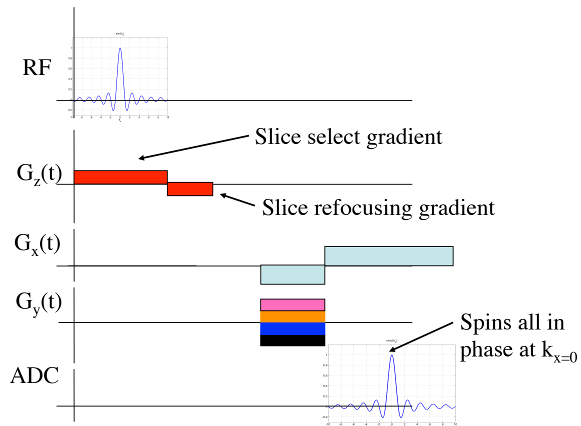
This has the form of an inverse Fourier transform, where we are integrating the contributions of the field $B_1(\tau)$ at the k - space point $k(\tau,t)$.



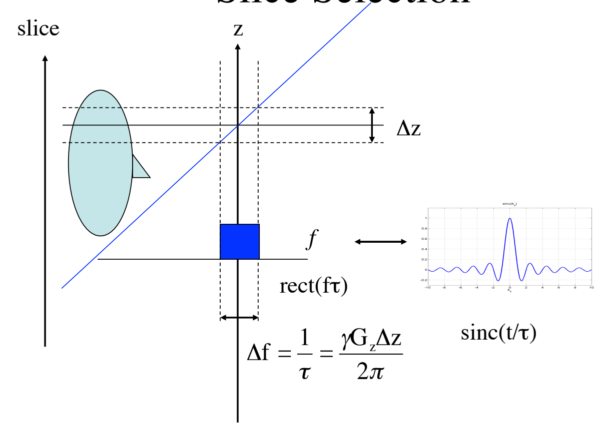
Slice Selection

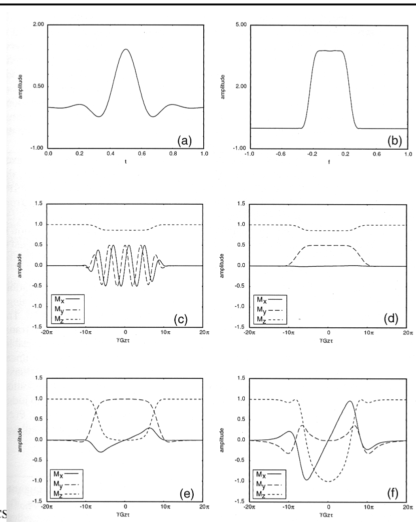


Gradient Echo



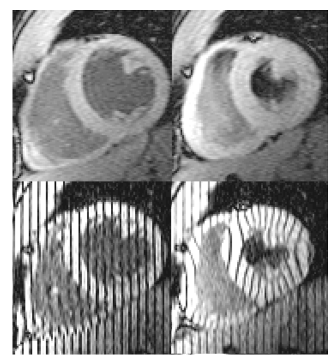
Slice Selection





Nishimura 1996

Cardiac Tagging



Multi-dimensional Excitation k-space

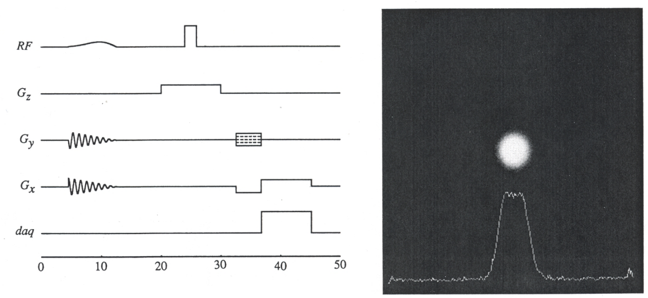
$$M_{xy}(t,\mathbf{r}) = jM_0\int_{-\infty}^{t}\omega_1(\tau)\exp\left(-j\gamma\int_{\tau}^{t}\mathbf{G}(s)\cdot\mathbf{r}\,ds\right)d\tau$$

$$= jM_0\int_{-\infty}^{t}\omega_1(\tau)\exp\left(j2\pi\mathbf{k}(\tau)\cdot\mathbf{r}\right)d\tau$$

where $\mathbf{k}(\tau) = -\frac{\gamma}{2\pi}\int_{\tau}^{t}\mathbf{G}(t')dt'$

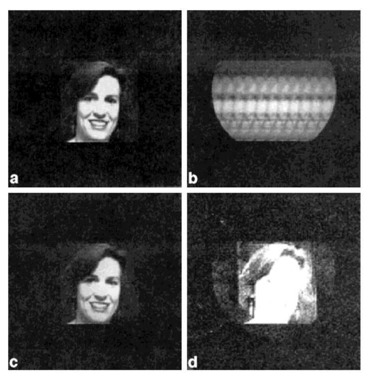
Pauly et al 1989

Excitation k-space



Pauly et al 1989

Excitation k-space



Panych MRM 1999

Example

$M_{xy}(x) = M_0\cos(4\pi x)$

$F\left(M_{xy}(x)\right) = \frac{M_0}{2}\left(\delta(k_x-2)+\delta(k_x+2)\right)$

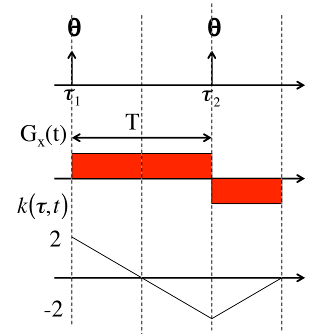
$g_{\max} = 4\ G/cm$

$\frac{\gamma}{2\pi}g_{\max}T = 4\ \text{cm}^{-1};\ \ \text{T} = 235\ \mu\text{sec}$

with small tip angle approximation --> $\theta = \frac{1}{2}$

Compare with $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \rightarrow \theta = \frac{\pi}{6} = 0.5236$

Question: Should we use $\theta = \frac{\pi}{4}$ instead?



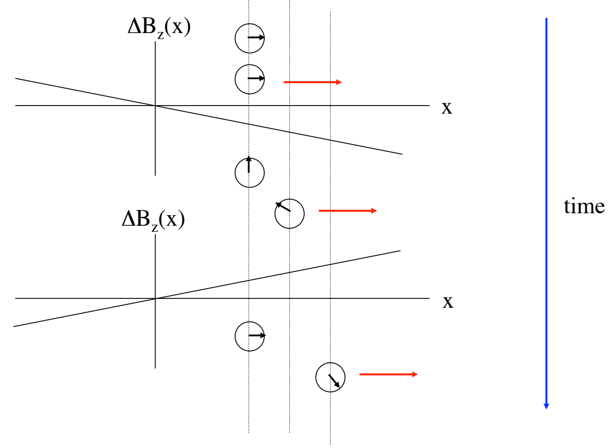
Moving Spins

So far we have assumed that the spins are not moving (aside from thermal motion giving rise to relaxation), and contrast has been based upon T_1, T_2, and proton density. We were able to achieve different contrasts by adjusting the appropriate pulse sequence parameters.

Biological samples are filled with moving spins, and we can also use MRI to image the movement. Examples: blood flow, diffusion of water in the white matter tracts. In addition, we can also sometimes induce motion into the object to image its mechanical properties, e.g. imaging of stress and strain with MR elastography.

TT. Liu, BE280A, UCSD Fall 2012

Phase of Moving Spin



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Phase of a Moving Spin

$$\begin{aligned}\varphi(t) &= -\int_0^t \Delta\omega(\tau)d\tau \\ &= -\int_0^t \gamma \Delta B(\tau)d\tau \\ &= -\int_0^t \gamma \vec{G}(\tau)\cdot\vec{r}(\tau)d\tau \\ &= -\gamma\int_0^t \left[G_x(\tau)x(\tau)+G_y(\tau)y(\tau)+G_z(\tau)z(\tau)\right]d\tau\end{aligned}$$

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Phase of Moving Spin

Consider motion along the x-axis

$$x(t) = x_0 + vt + \frac{1}{2}at^2$$

$$\begin{aligned}\varphi(t) &= -\gamma\int_0^t G_x(\tau)x(\tau)d\tau \\ &= -\gamma\int_0^t G_x(\tau)\left[x_0 + v\tau + \frac{1}{2}a\tau^2\right]d\tau \\ &= -\gamma\left[x_0\int_0^t G_x(\tau)d\tau + v\int_0^t G_x(\tau)\tau d\tau + \frac{a}{2}\int_0^t G_x(\tau)\tau^2 d\tau\right] \\ &= -\gamma\left[x_0 M_0 + vM_1 + \frac{a}{2}M_2\right]\end{aligned}$$

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Phase of Moving Spin

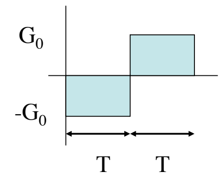
$$\varphi(t) = -\gamma\left[x_0 M_0 + vM_1 + \frac{a}{2}M_2\right]$$

$M_0 = \int_0^t G_x(\tau)d\tau$ Zeroth order moment

$M_1 = \int_0^t G_x(\tau)\tau d\tau$ First order moment

$M_2 = \int_0^t G_x(\tau)\tau^2 d\tau$ Second order moment

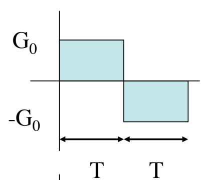
Flow Moment Example



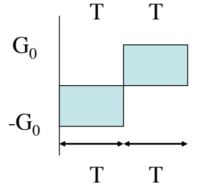
$$M_0 = \int_0^t G_x(\tau)d\tau = 0$$

$$\begin{aligned} M_1 &= \int_0^t G_x(\tau)\tau d\tau \\ &= -\int_0^T G_0 \tau d\tau + \int_T^{2T} G_0 \tau d\tau \\ &= G_0\left[-\frac{\tau^2}{2}\Big|_0^T + \frac{\tau^2}{2}\Big|_T^{2T}\right] \\ &= G_0\left[-\frac{T^2}{2} + \frac{4T^2}{2} - \frac{T^2}{2}\right] = G_0 T^2 \end{aligned}$$

Phase Contrast Angiography (PCA)



$$\varphi_1 = -\gamma v_x M_1 = \gamma v_x G_0 T^2$$



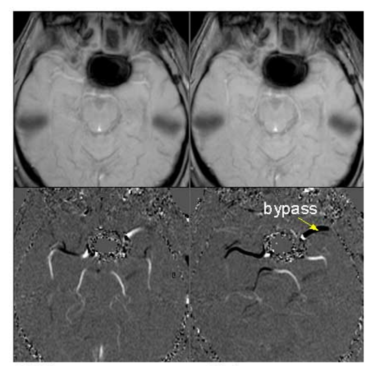
$$\varphi_2 = -\gamma v_x M_1 = -\gamma v_x G_0 T^2$$

$$\Delta\varphi = \varphi_1 - \varphi_2 = 2\gamma v_x G_0 T^2$$

$$v_x = \frac{\Delta\varphi}{2G_0T^2}$$

PCA example

$-G_0$



White = Flow direction AP (↓)

White = Flow direction RL (→)

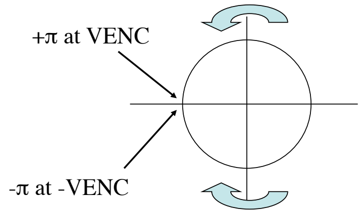
http://www.medical.philips.com/main/products/mri/assets/images/case_of_week/cotw_51_s5.jpg

Aliasing in PCA

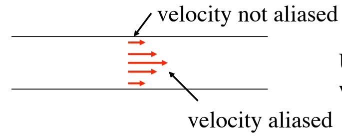
Define VENC as the velocity at at which the phase is 180 degrees.

$$VENC \equiv \frac{\pi}{\gamma G_0 T^2}$$

Because of phase wrapping the velocity of spins flowing faster than VENC is ambiguous.



Aliasing Solutions



Use data from regions with slower flow

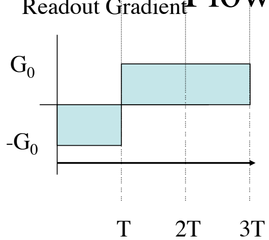
Use multiple VENC values so that the phase differences are smaller than π radians.

$$\varphi_1 = \pi \frac{v_x}{VENC_1}$$

$$\varphi_2 = \pi \frac{v_x}{VENC_2}$$

$$\varphi_1 - \varphi_2 = \pi v_x \left(\frac{1}{VENC_1} - \frac{1}{VENC_2} \right)$$

Flow Artifacts



During readout moving spins within the object will accumulate phase that is in addition to the phase used for imaging. This leads to

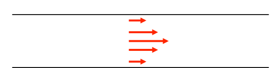
1) Net phase at echo time TE = 2T.
2) An apparent shift in position of the object.
3) Blurring of the object due to a quadratic phase term.

Flow Artifacts

Plug Flow

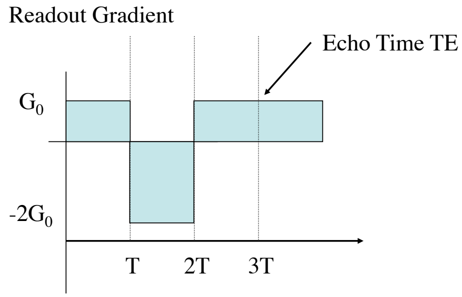
All moving spins in the voxel experience the same phase shift at echo time.

Laminar Flow



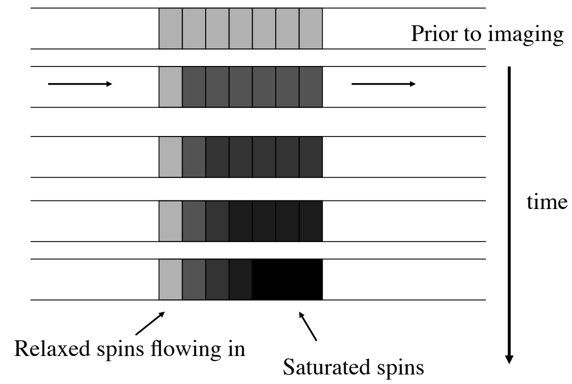
Spins have different phase shifts at echo time. The dephasing causes the cancelation and signal dropout.

Flow Compensation

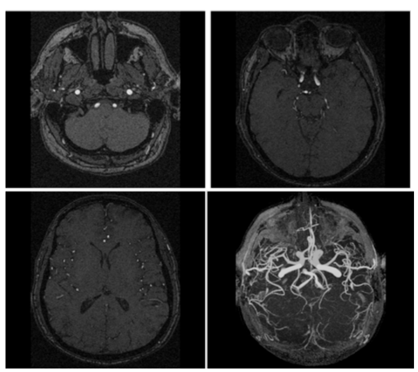


At TE both the first and second order moments are zero, so both stationary and moving spins have zero net phase.

Inflow Effect



Time of Flight Angiography

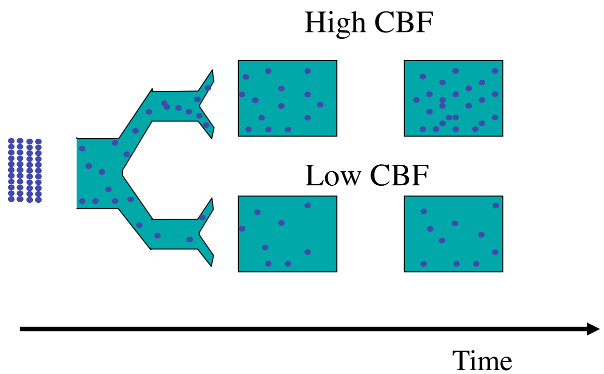


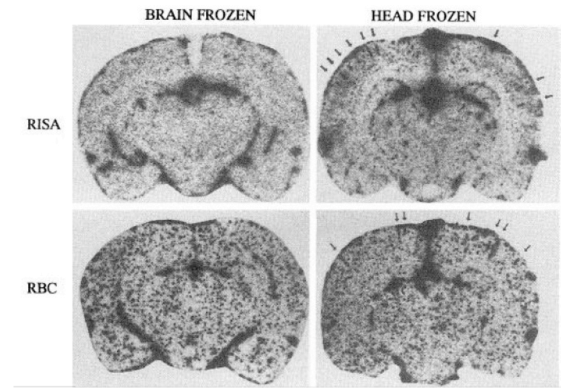
Cerebral Blood Flow (CBF)

CBF = Perfusion
= Rate of delivery of arterial blood to a capillary bed in tissue.

Units: $\dfrac{\text{(ml of Blood)}}{\text{(100 grams of tissue)(minute)}}$

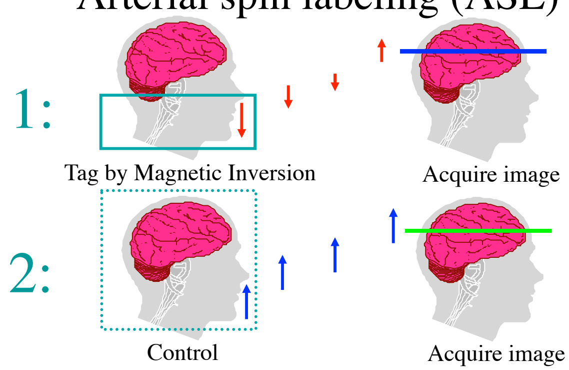
Typical value is 60 ml/(100g-min) or 60 ml/(100 ml-min) = 0.01 s^{-1}, assuming average density of brain equals 1 gm/ml





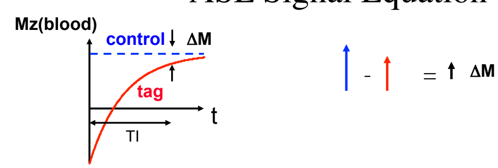
Bereczki et al 1992

Arterial spin labeling (ASL)



Control - Tag = $\Delta M \propto$ CBF

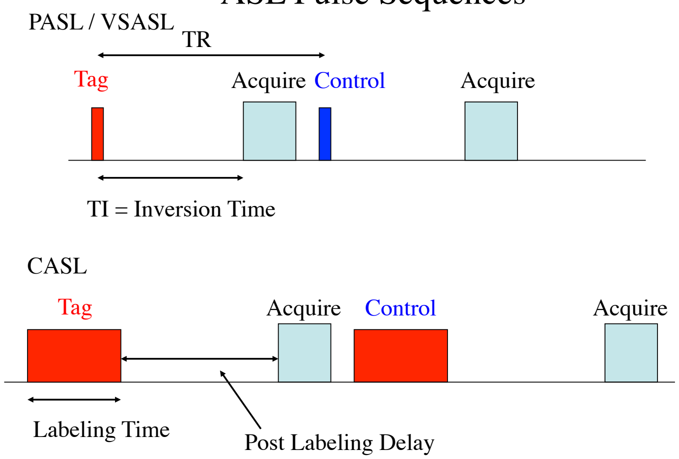
ASL Signal Equation



$$\Delta M = CBF \cdot A_{eff}$$

A_{eff} is the effective area of the arterial bolus. It depends on both physiology and pulse sequence parameters.

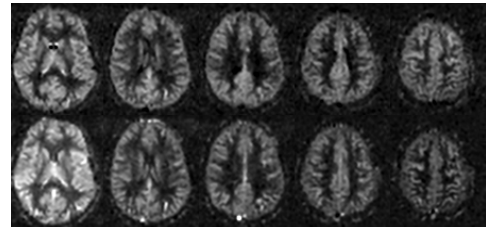
ASL Pulse Sequences



Multislice CASL and PICORE

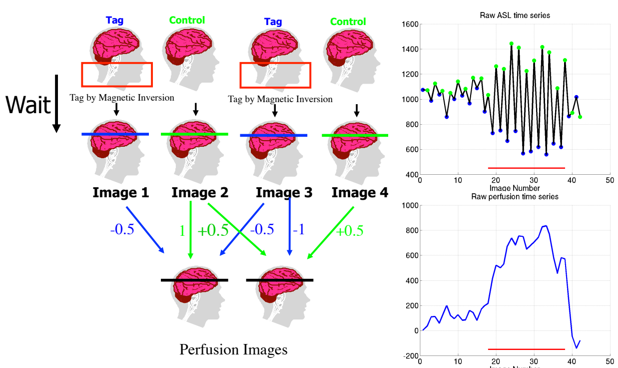
CASL

PICORE QUIPSS II

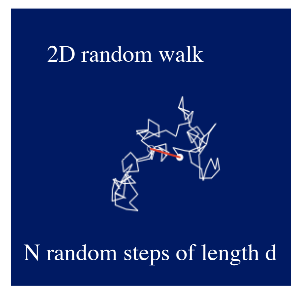


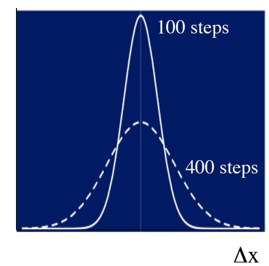
Credit: E. Wong

ASL Time Series



Diffusion





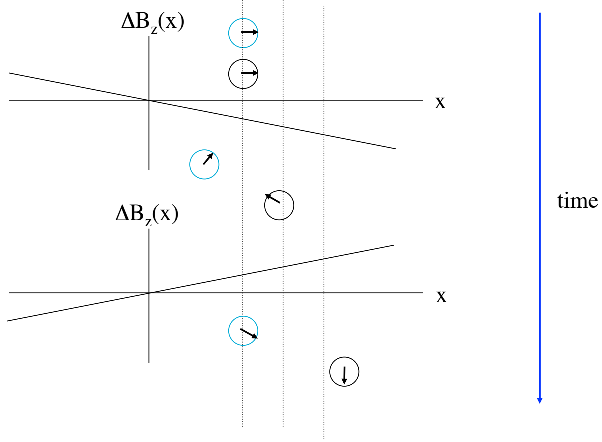
$<\Delta x^2> = Nd^2 = 2DT$

D = diffusivity

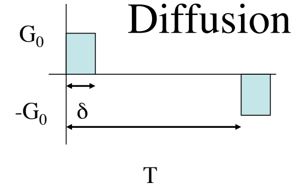
In brain:
$D \cong 0.001\ mm^2/s$
For T=100 msec,
$\Delta x \cong 15\ \mu$

Credit: Larry Frank

Diffusing Spins



Diffusion Weighting



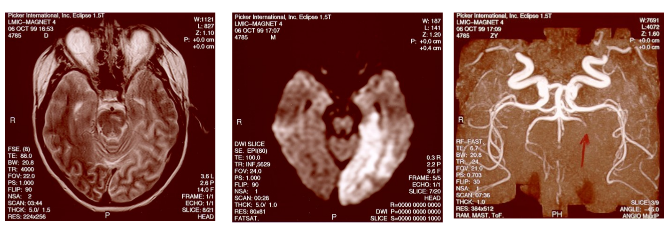
Signal

$$S \propto e^{-\gamma^2 G_0^2 \delta^2 DT} = e^{-bD} \quad \text{where } b = \gamma^2 G_0^2 \delta^2 (T - \delta/3)$$

Diffusivity

Diffusion Weighted Images

T2 weighted　　Diffusion Weighted　　Angiogram

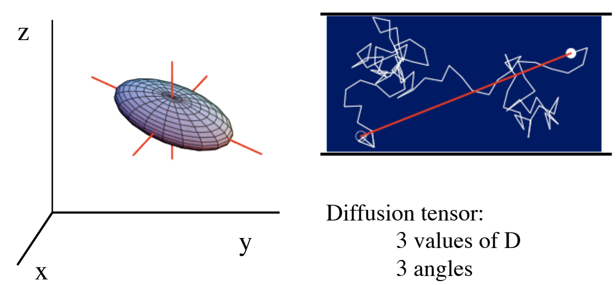


After a stroke, normal water movement is restricted in the region of damage. Diffusivity decreases, so the signal intensity increases.

http://lehighmri.com/cases/dwi/patient-b.html

Restricted Diffusion

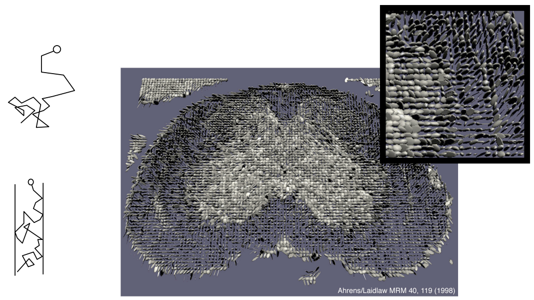
D depends on direction



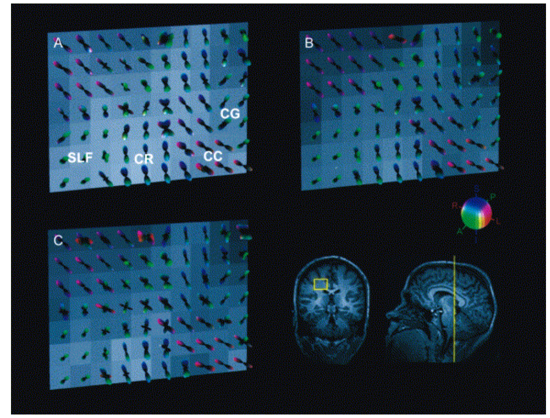
Diffusion tensor:
- 3 values of D
- 3 angles

Credit: Larry Frank

Diffusion Imaging Example



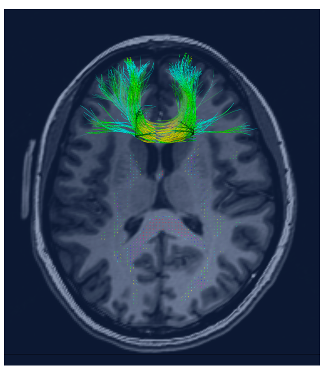
Q-ball imaging

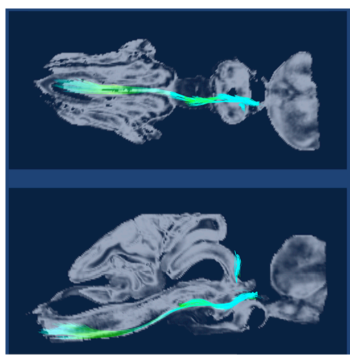


Tuch et al, Neuron 2003

TT. Liu, BE280A, UCSD Fall 2012

Fiber tract mapping of neural connectivity





Courtesy of L. Frank

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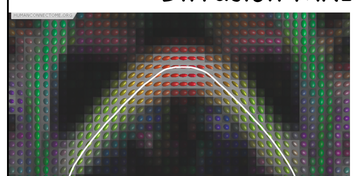
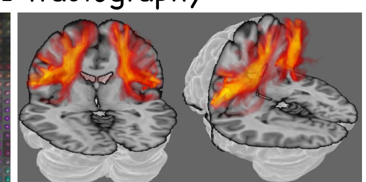
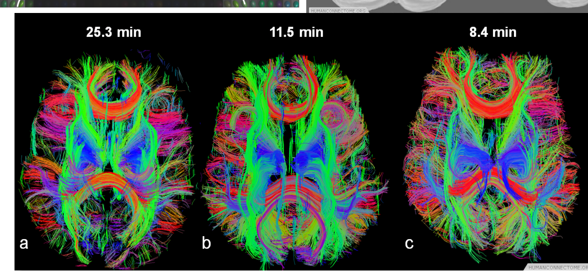
Diffusion MRI Tractography







from the Human Connectome Project

TT. Liu, BE280A, UCSD Fall 2012