BENG280A, Principles of Biomedical Imaging 10/23/13
Fall Quarter 2013

HOMEWORK #4

Due at 5 pm on Wednesday 10/30/13

Homework policy: Homeworks can be turned in during class or to the TA's mailbox in the Graduate Student Lounge. Late homeworks will be marked down by 20% per day. If you know that you need to turn in a homework late because of an emergency or academic travel, please let the TA know ahead of time. Collaboration is encouraged on homework assignments, however, the homework that you submit should reflect your own understanding of the material.

Readings: Review the class notes.

Problems:

1. Let $G(k,\theta)$ be the 1-D Fourier transform of the projection $g(l,\theta)$.
   a) Show that $g(l,\theta+\pi) = g(-l,\theta)$
   b) Next, show that $G(k,\theta+\pi) = G(-k,\theta)$
   c) Use your results to show that $\int_0^{2\pi}\int_0^{\infty} G(k,\theta)e^{j2\pi(xk\cos\theta+yk\sin\theta)}kdkd\theta$ can be written as

   $$\int_0^{\pi}\int_{-\infty}^{\infty} G(k,\theta)e^{j2\pi k(x\cos\theta+y\sin\theta)}|k|dkd\theta$$

2. Consider the CT k-space filter $G(k) = |k|w(k)$ where $w(k)$ is a windowing function. For each of the following window functions, use MATLAB to plot the k-space filter. Also, derive analytical expressions for the inverse Fourier transforms of the window functions.

   a) The Ram-Lak Filter with $w(k) = rect\left(\frac{k}{2k_{\max}}\right)$.

   b) A Hanning window defined as $w(k) = rect\left(\frac{k}{2k_{\max}}\right)\left(0.5+0.5\cos\left(\frac{\pi k}{k_{\max}}\right)\right)$.

   c) Use MATLAB to plot out and compare the inverse transforms with the expressions you derived in parts (a) and (b). Comment on the relative advantages and disadvantages of the two filters for CT reconstruction.

3. Consider the object $f(x,y) = 2rect(x/2)rect(y/2)\cos(4\pi x+2\pi y)$ that you looked at in Homework 3. Use your results from that HW3 to help you answer the following questions:
   a) Derive and sketch the projections at 0 degrees and 90 degrees. Justify your answer.
   b) Determine the angle that has the maximum projection. Provide an explicit expression and sketch the projection at this angle.

4. A parallel beam CT imaging system is used to image an object defined as:

   $$f(x,y) = rect(x,y) + \Big(rect(x,y) ** \big[(\delta(x-2)+\delta(x+2))\delta(y)\big] ** \big[(\delta(y-4)+\delta(y+4))\delta(x)\big]\Big)$$

   a) Sketch the object and draw the projections of the object at 0 degrees and 45 degrees.
   b) Derive the Fourier transform of the object
   c) Show that the Projection-slice theorem holds for the projections at 0 and 45 degrees.

MATLAB exercises on next page

Matlab Exercise 1: In this exercise you will use MATLAB to plot out the complex function $g(x) = \exp(j2\pi k_x x)$.

Enter in the following MATLAB commands (you will want to save this in a script, so you can easily modify):

```
j = sqrt(-1);
x = -10:.1:10;
kx = .1;
g = exp(j*kx*2*pi*x);
figure(1);clf;
quiver3(x,zeros(size(x)),zeros(size(x)),zeros(size(x)),real(g),imag(g));
```

1) Describe the main features of the plot and explain the main features – i.e. what is the period of repetition and the "handedness of the helix". You will find it useful to use the "rotate" button on the MATLAB plotting interface to look at the helix from different views.
2) Which way do the arrows point at x = 0, x = 2.5 and x = 5.0? Explain how the directions match up with the analytical expressions of the function.
3) What happens when you make `kx = -0.1`? Explain what is going on.
4) What happens when you make `kx = 0.2`? Explain what you observe and compare this to the original plot (e.g. from part 1).

Next enter in the following commands:

```
span = -10:.5:10;
[x,y] = meshgrid(span,span);
kx = .1;ky = .1;
g = exp(j*2*pi*(kx*x + ky*y));
figure(2);clf;
quiver(x,y,real(g),imag(g));
grid;
```

1) Explain what you are seeing. What function are we plotting? How does the way in which we are plotting the function differ from the way we did it in the first half of this exercise?
2) What is the fundamental period of the object – does this match the analytical prediction?
3) Which way do the arrows point at (0,0), (5,5), and (2.5, 2.5)? Do these match the analytical prediction?
4) How does the plot change when you make kx = -0.1? Explain the differences.
5) How does the plot change when you make kx = 0.05? Explain the differences.
6) Try your own combination of values of kx and ky (different from what was done above) and explain the main features of what you are seeing.