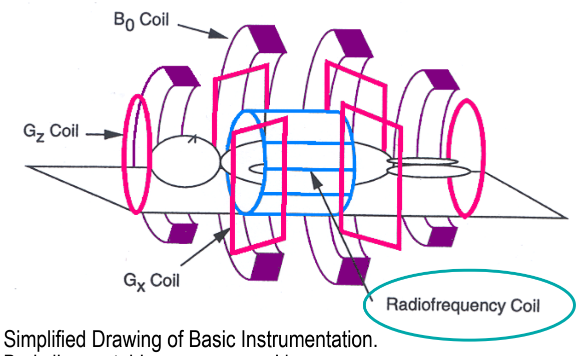
Bioengineering 280A
Principles of Biomedical Imaging

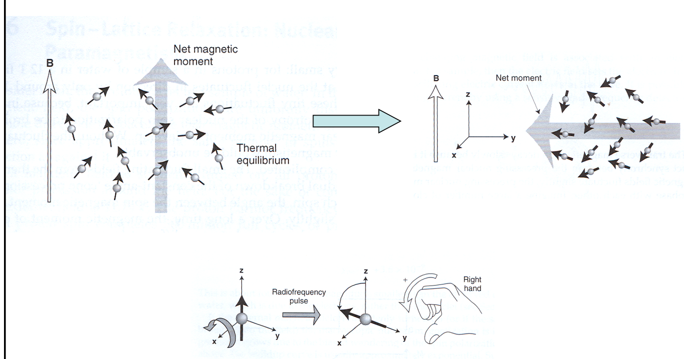
Fall Quarter 2013
MRI Lecture 4



Simplified Drawing of Basic Instrumentation.
Body lies on table encompassed by
coils for static field B_o,
gradient fields (two of three shown),
and radiofrequency field B_1.

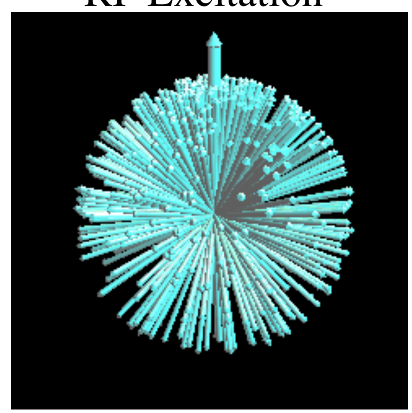
Image, caption: copyright Nishimura, Fig. 3.15

RF Excitation



From Levitt, Spin Dynamics, 2001

RF Excitation



http://www.drcmr.dk/main/content/view/213/74/

RF Excitation

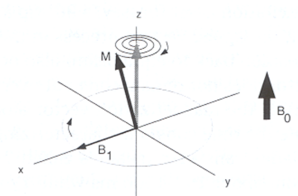


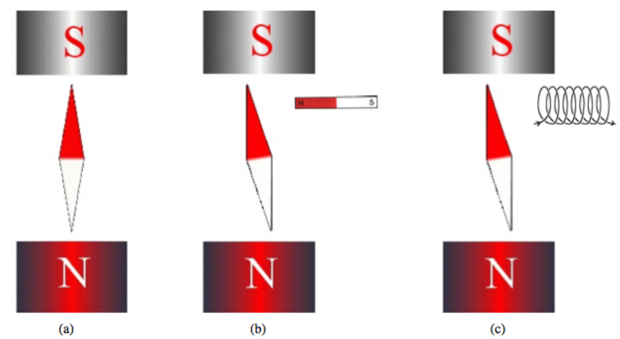
Image & caption: Nishimura, Fig. 3.2

At equilibrium, net magnetizaion is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

B_1 radiofrequency field tuned to Larmor frequency and applied in transverse (*xy*) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the *z*-axis.
- lab frame of reference

http://www.eecs.umich.edu/%7Ednoll/BME516/

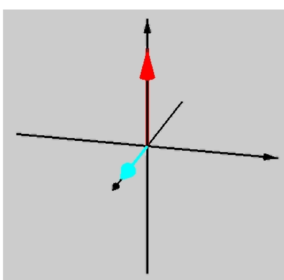
On-Resonance Excitation

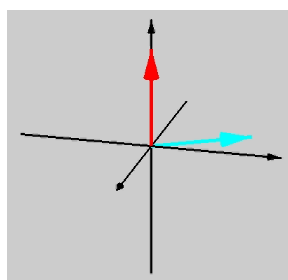


Hanson 2009

http://www.drcmr.dk/JavaCompass/

RF Excitation

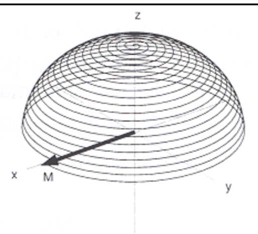




http://www.eecs.umich.edu/%7Ednoll/BME516/

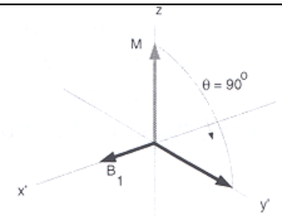
Rotating Frame of Reference

Reference everything to the magnetic field at isocenter.

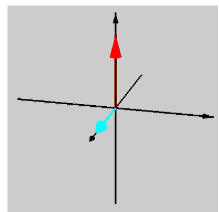


a) Laboratory frame behavior of **M**

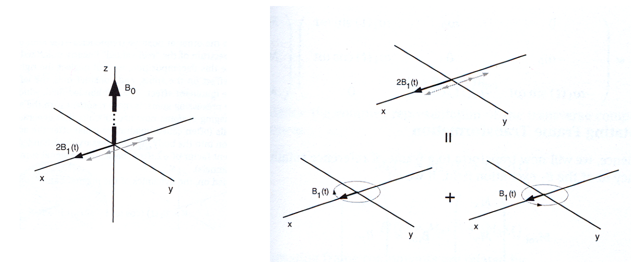
Images & caption: Nishimura, Fig. 3.3



b) Rotating frame behavior of **M**

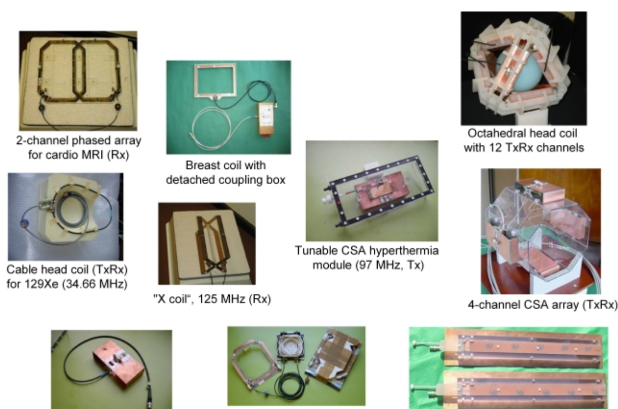


http://www.eecs.umich.edu/%7Ednoll/BME516/



$$\mathbf{B}_1(t) = 2B_1(t)\cos(\omega t)\mathbf{i}$$
$$= B_1(t)\big(\cos(\omega t)\mathbf{i} - \sin(\omega t)\mathbf{j}\big) + B_1(t)\big(\cos(\omega t)\mathbf{i} + \sin(\omega t)\mathbf{j}\big)$$

Nishimura 1996



2-channel phased array for cardio MRI (Rx)

Breast coil with detached coupling box

Octahedral head coil with 12 TxRx channels

Cable head coil (TxRx) for 129Xe (34.66 MHz)

"X coil", 125 MHz (Rx)

Tunable CSA hyperthermia module (97 MHz, Tx)

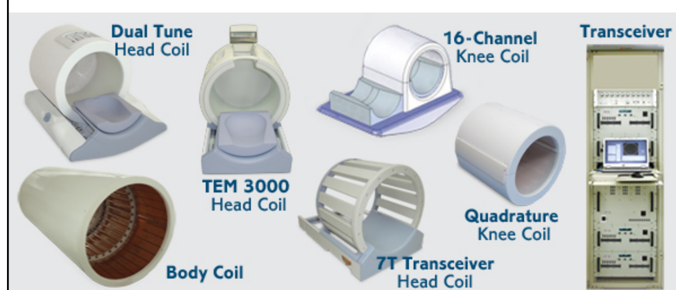
4-channel CSA array (TxRx)

125-MHz CSA

Lung coil (TxRx) with differential transmitter for 129Xe (34.66 MHz)

2-channel strip-line array (TxRx)

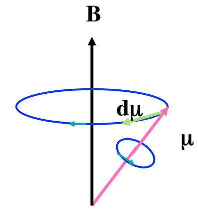
http://www.berlin.ptb.de/en/org/8/81/811/ResearchTopics/CoilDevelopment.html



http://www.mrinstruments.com/

Precession

$$\frac{d\mu}{dt} = \mu \times \gamma \mathbf{B}$$

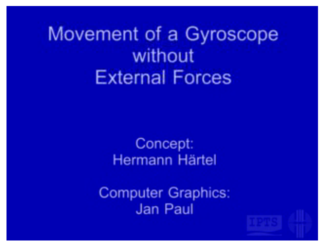


Analogous to motion of a gyroscope

Precesses at an angular frequency of

$$\omega = \gamma B$$

This is known as the **Larmor** frequency.



http://www.astrophysik.uni-kiel.de/~hhaertel/mpg_e/gyros_free.mpg

Rotating Frame Bloch Equation

$$\frac{d\mathbf{M}_{rot}}{dt} = \mathbf{M}_{rot} \times \gamma \mathbf{B}_{eff}$$

$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}; \quad \omega_{rot} = \begin{bmatrix} 0 \\ 0 \\ -\omega \end{bmatrix}$$

Note: we use the RF frequency to define the rotating frame. If this RF frequency is on-resonance, then the main B0 field doesn't cause any precession in the rotating frame. However, if the RF frequency is off-resonance, then there will be a net precession in the rotating frame that is give by the difference between the RF frequency and the local Larmor frequency.

$$Let \ \mathbf{B}_{rot} = B_1(t)\mathbf{i} + B_0\mathbf{k}$$

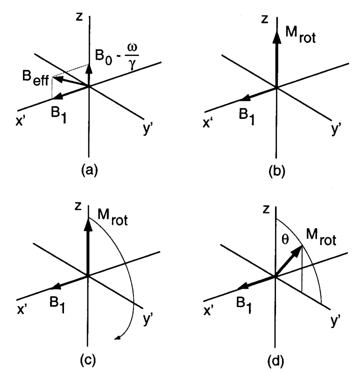
$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}$$

$$= B_1(t)\mathbf{i} + \left(B_0 - \frac{\omega}{\gamma}\right)\mathbf{k}$$

If $\omega = \omega_0$

$$= \gamma B_0$$

Then $\mathbf{B}_{eff} = B_1(t)\mathbf{i}$



Flip angle

$$\theta = \int_0^{\tau} \omega_1(s)\,ds$$

where

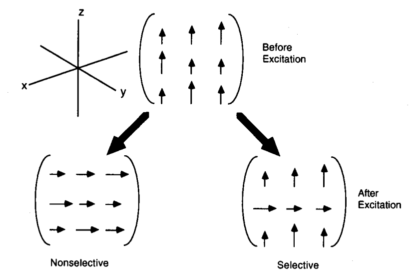
$$\omega_1(t) = \gamma B_1(t)$$

Nishimura 1996

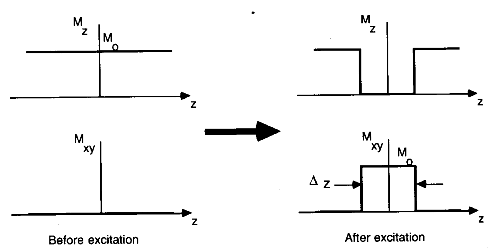
Example

$\tau = 400\ \mu\text{sec};\ \theta = \pi/2$

$$B_1 = \frac{\theta}{\gamma\tau} = \frac{\pi/2}{2\pi(4257 Hz/G)(400e-6)} = 0.1468\ G$$



Nishimura 1996



Nishimura 1996

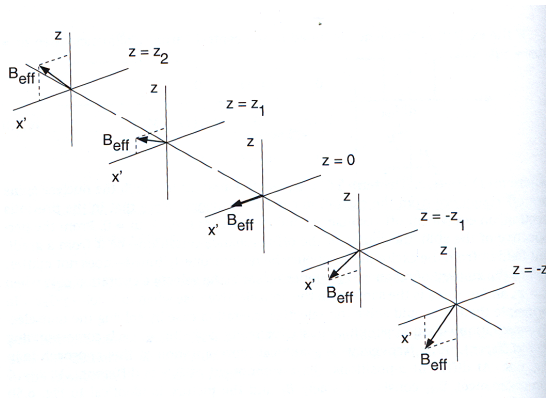
$$Let\ \mathbf{B}_{rot} = B_1(t)\mathbf{i} + \left(B_0 + \gamma G_z z\right)\mathbf{k}$$

$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}$$

$$= B_1(t)\mathbf{i} + \left(B_0 + \gamma G_z z - \frac{\omega}{\gamma}\right)\mathbf{k}$$

If $\omega = \omega_0$

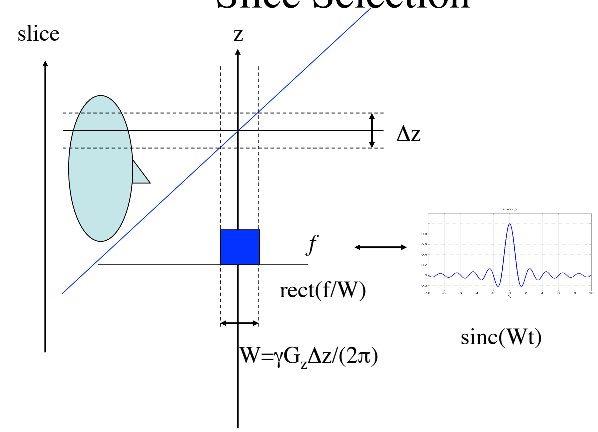
$$\mathbf{B}_{eff} = B_1(t)\mathbf{i} + \left(\gamma G_z z\right)\mathbf{k}$$



TT. Liu, BE280A, UCSD Fall 2013

Nishimura 1996

Slice Selection



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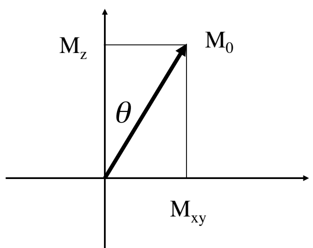
Small Tip Angle Approximation



For small θ

$$M_z = M_0 \cos\theta \approx M_0$$

$$M_{xy} = M_0 \sin\theta \approx M_0\theta$$

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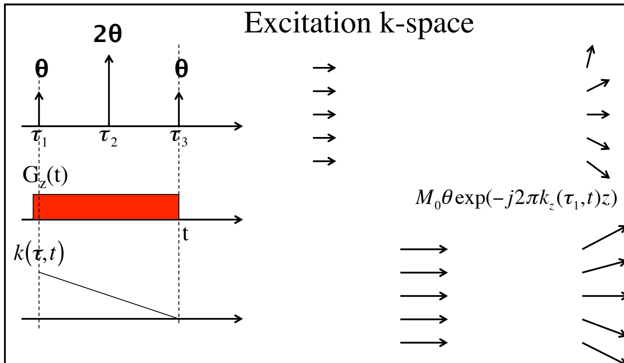
Excitation k-space



$$k(\tau,t) = \frac{\gamma}{2\pi}\int_{\tau}^{t} G_z(s)\,ds$$

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Excitation k-space

At each time increment of width $\Delta\tau$, the excitation $B_1(\tau)$ produces an increment in magnetization of the form $\Delta M_{xy} \approx jM_0\gamma B_1(\tau)\Delta\tau$ (small tip angle approximation)

In the presence of a gradient, this will accumulate phase of the form $\varphi = -\gamma\int_\tau^t zG_z(s)\,ds$, such that the incremental magnetization at time t is

$$\Delta M_{xy}(t,z\,;\,\tau) = jM_0\gamma B_1(\tau)\exp\left(-j\gamma\int_\tau^t zG_z(s)\,ds\right)\Delta\tau$$

Integrating over all time increments, we obtain

$$M_{xy}(t,z) = jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp\left(-j\gamma\int_\tau^t zG_z(s)\,ds\right)d\tau$$

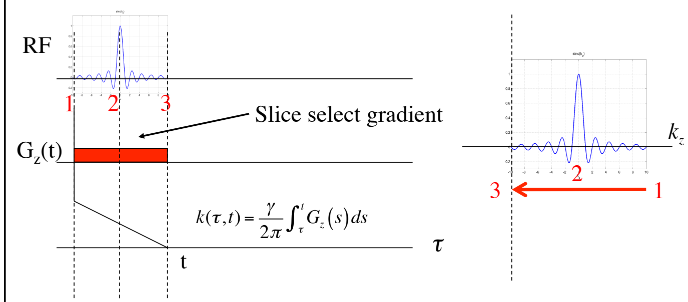
$$= jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp\left(-j2\pi k(\tau,t)z\right)d\tau$$

where $k(\tau,t) = \frac{\gamma}{2\pi}\int_\tau^t G_z(s)\,ds$

Excitation k-space

$$M_{xy}(t,z) = jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp\left(-j2\pi k(\tau,t)z\right)d\tau$$

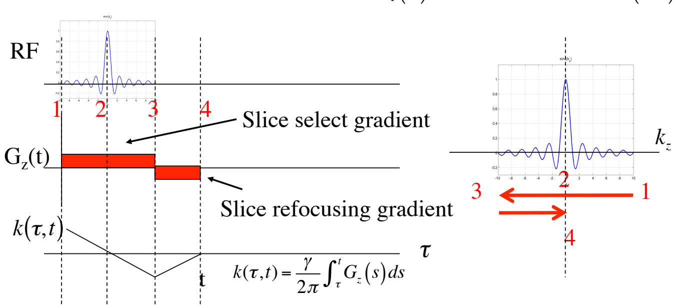
This has the form of a Fourier transform, where we are integrating the contributions of the field $B_1(\tau)$ at the k-space point $k(\tau,t)$.



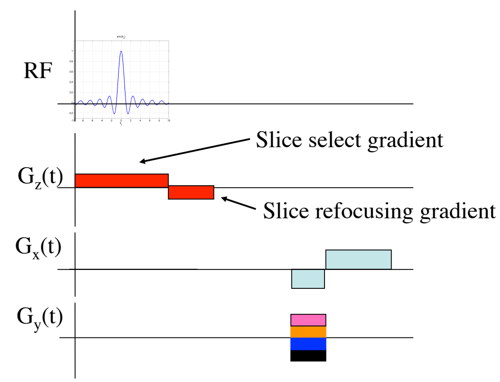
Refocusing

$$M_{xy}(t,z) = jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp\left(-j2\pi k(\tau,t)z\right)d\tau$$

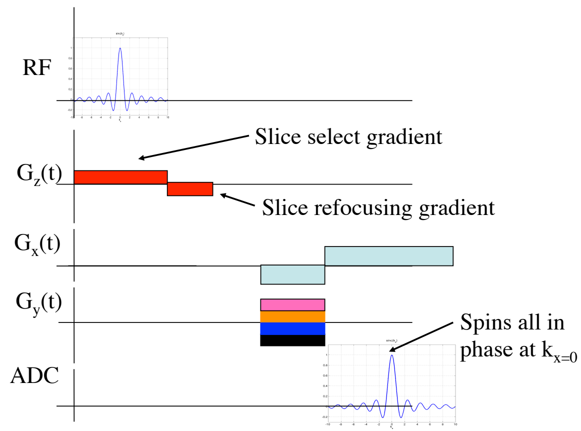
This has the form of a Fourier transform, where we are integrating the contributions of the field $B_1(\tau)$ at the k-space point $k(\tau,t)$.



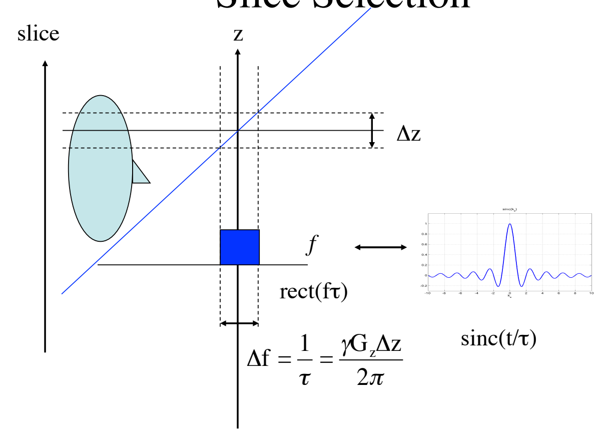
Slice Selection

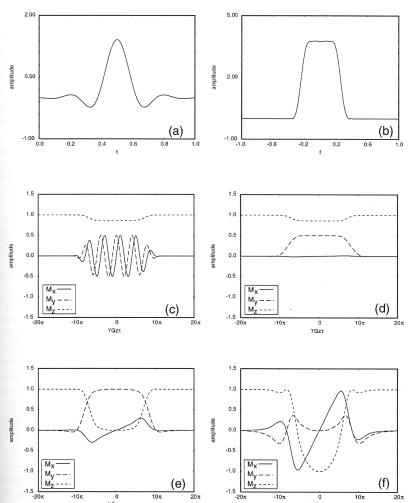


Gradient Echo



Slice Selection





Nishimura 1996

Example

$M_{xy}(x) = M_0 \cos(4\pi x)$

$F\left(M_{xy}(x)\right) = \frac{M_0}{2}\left(\delta(k_x - 2) + \delta(k_x + 2)\right)$

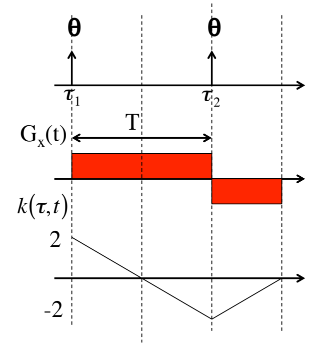
$g_{\max} = 4\ G/cm$

$\frac{\gamma}{2\pi} g_{\max} T = 4\ \text{cm}^{-1};\ \ \text{T} = 235\ \mu\text{sec}$

with small tip angle approximation --> $\theta = \frac{1}{2}$

Compare with $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \rightarrow \theta = \frac{\pi}{6} = 0.5236$

Question: Should we use $\theta = \frac{\pi}{4}$ instead?



Exercise: Sketch the quiver diagrams corresponding to the contributions of the two RF pulses and show that they produce the desired pattern.

Multi-dimensional Excitation k-space

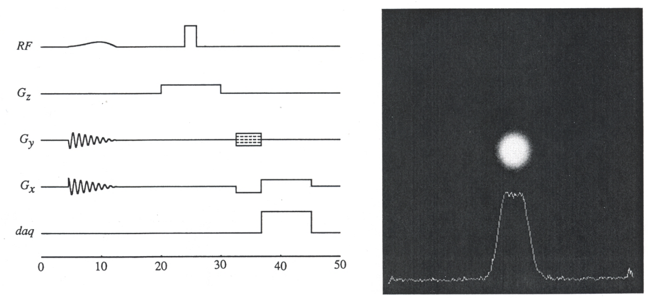
$$M_{xy}(t,\mathbf{r}) = jM_0\int_{-\infty}^{t}\omega_1(\tau)\exp\left(-j\gamma\int_{\tau}^{t}\mathbf{G}(s)\cdot\mathbf{r}\,ds\right)d\tau$$

$$= jM_0\int_{-\infty}^{t}\omega_1(\tau)\exp\left(j2\pi\mathbf{k}(\tau)\cdot\mathbf{r}\right)d\tau$$

where $\mathbf{k}(\tau) = -\frac{\gamma}{2\pi}\int_{\tau}^{t}\mathbf{G}(t')\,dt'$

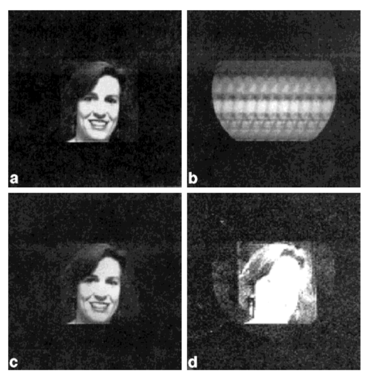
Pauly et al 1989

Excitation k-space



Pauly et al 1989

Excitation k-space



Panych MRM 1999

Cardiac Tagging