Image Quality
Lecture 2

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Resident Physics Course
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Topics

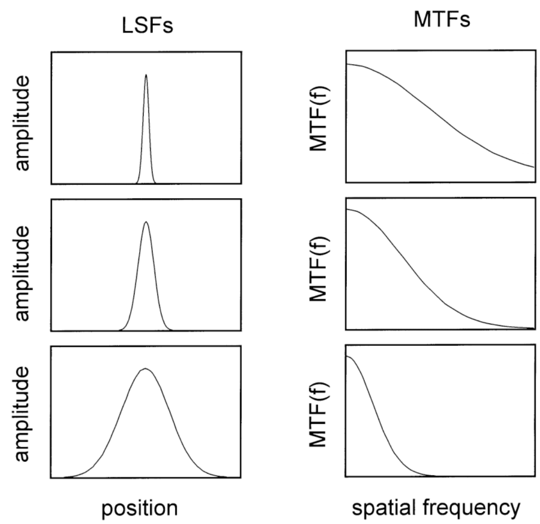
Review MTF question

Noise

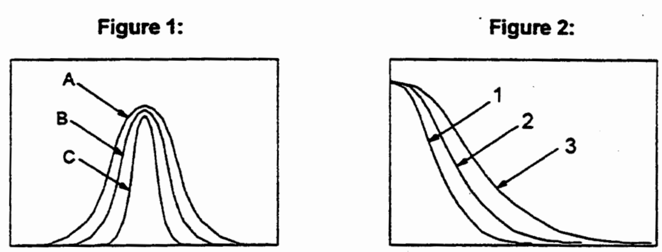
Receiver Operating Characteristics

Sampling and Aliasing

MTF = Fourier Transform (LTF)



Bushberg et al 2001



10. Referring to Figure 1 which shows LSFs, and Figure 2 which shows the corresponding modulation transfer functions (MTFs), which MTF corresponds to LSF C?

A. MTF number 1
B. MTF number 2
C. MTF number 3

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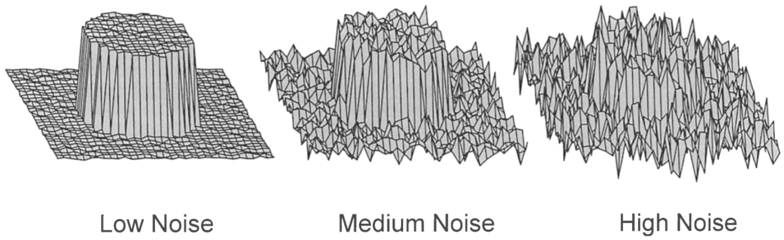
11. Referring to Figure 2 illustrating MTFs, the axes should be labeled _____ for the y-axis and ______ for the x-axis.

A. Relative amplitude, distance (mm)
B. Spatial frequency (lp/mm), distance (mm)
C. Lateral dimension (mm), Fresnel ratio
D. Relative amplitude, spatial frequency (lp/mm)
E. Relative amplitude, relative amplitude

Noise and Image Quality



Bushberg et al 2001

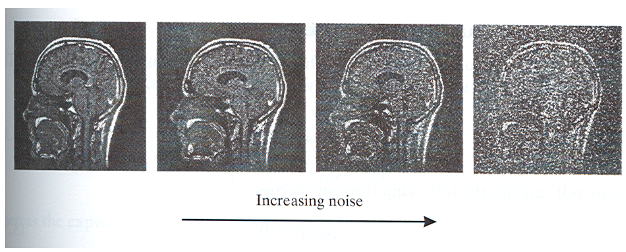


Figure 3.10
The effect of noise on image quality: image quality decreases rapidly with increasing noise contamination.

Prince and Links 2005

What is Noise?

Fluctuations in either the imaging system or the object being imaged.

Quantization Noise: Due to conversion from analog waveform to digital number.

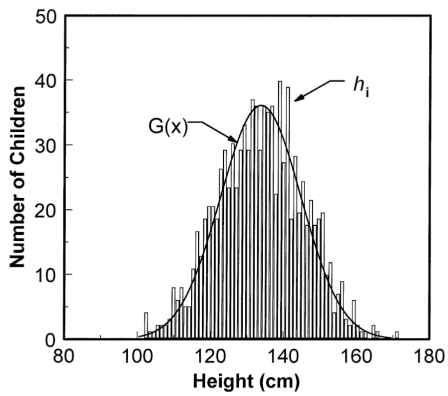
Quantum Noise: Random fluctuation in the number of photons emitted and recorded.

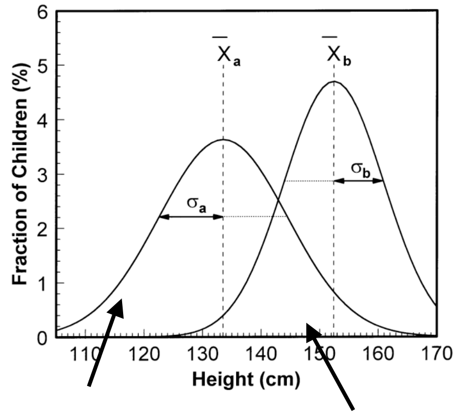
Thermal Noise: Random fluctuations present in all electronic systems. Also, sample noise in MRI

Other types: flicker, burst, avalanche - observed in semiconductor devices.

Structured Noise: physiological sources, interference

Histograms and Distributions

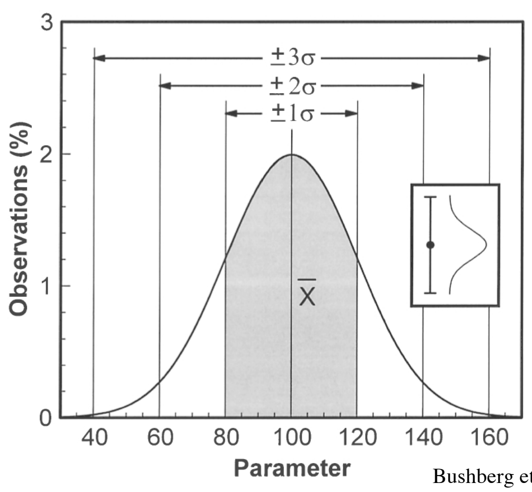




3rd grade heights 6th grade heights

Bushberg et al 2001

Gaussian Distribution



Bushberg et al 2001

1, 2, and 3 standard deviation intervals correspond to 68%, 95%, and 99% of the observations

Poisson Process

Events occur at random instants of time at an average rate of λ events per second.

Examples: arrival of customers to an ATM, emission of photons from an x-ray source, lightning strikes in a thunderstorm.

$\lambda =$ Average rate of events per second

$\lambda t =$ Average number of events at time t

$\lambda t =$ Variance in number of events

Quantum Noise

For a Poisson process, the mean = variance, i.e. $\overline{X} = \sigma^2$

Therefore, the standard deviation is given by $\sigma = \sqrt{\overline{X}}$

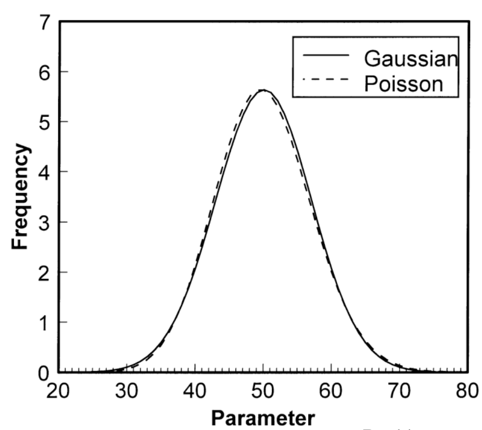
For X-ray systems, if the mean number of counts is N, then the standard deviation in the number of counts is $\sigma = \sqrt{N}$.

$$SNR = \frac{N}{\sigma} = \sqrt{N}.$$

TABLE 10-1. EXAMPLES OF NOISE VERSUS PHOTONS

Photons/Pixel (N)	Noise (σ) ($\sigma = \sqrt{N}$)	Relative Noise (σ/N) (%)	SNR (N/σ)
10	3.2	32	3.2
100	10	10	10
1,000	31.6	3.2	32
10,000	100	1.0	100
100,000	316.2	0.3	316

SNR, signal-to-noise ratio.



Bushberg et al 2001

Poisson Distribution describes x-ray counting statistics.

Gaussian distribution is good approximation to Poisson when $\sigma = \sqrt{\bar{X}}$

G79. A series of measurements has a mean of 100 counts. A range of $\pm\sigma$ is _____ .

A. 95–105
B. 90–100
C. 68–137
D. 50–150
E. 33–167

G80. To achieve a standard deviation of 2%, _____ counts must be collected.

A. 400
B. 1,414
C. 2,500
D. 10,000
E. 40,000

G79. B The standard deviation σ is the square root of the mean, in this case $\sqrt{100} = 10$. There is a 68% probability that any random reading will fall within σ of the mean, and a 95% probability that it will fall within 2σ of the mean.

G80. C The percent standard deviation, $\%\sigma = (\sigma / N) \times 100 = (\sqrt{N} / N) \times 100 = 100 / \sqrt{N}$. In this case $100 / \sqrt{N} = 2$, so $N = 2500$.

G73. A radioactive sample is counted many times, and the mean is 2500 counts. 96% of the readings will lie between _____ and _____ counts.

A. 2300 2500
B. 2400 2500
C. 2400 2600
D. 2450 2550
E. 2500 2700

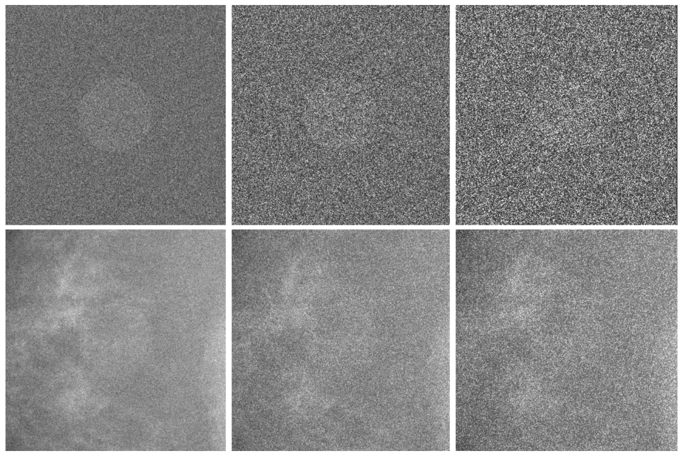
G73. C If a large number of measurements are made, approximately 67% will fall between $\pm\sigma$, and 96% between $\pm 2\sigma$ of the mean. The standard deviation $\sigma = \sqrt{N}$, or 50 in this case. $2500 \pm (2 \times \sigma) = 2400 - 2600$.

D70. How many counts must be collected in an instrument with zero background to obtain an error limit of 1% with a confidence interval of 95%?

A. 1000
B. 3162
C. 10,000
D. 40,000
E. 100,000

D70. D A 95% confidence interval means the counts must fall within two standard deviations (SD) of the mean (N).
Error limit = 1% = 2 SD/N, but SD = $N^{1/2}$.
Thus $0.01 = 2(N^{1/2})/N = 2/N^{1/2}$.
$[0.01]^2 = 4/N$.
N = 40,000.

Contrast Resolution



Bushberg et al 2001

Lower row shows effect of *structure noise*

Image Quality, T.T. Liu, Spring 2006

THE 2 × 2 DECISION MATRIX

	Actually Abnormal	Actually Normal
Diagnosed as Abnormal	True Positive (TP)	False Positive (FP)
Diagnosed as Normal	False Negative (FN)	True Negative (TN)

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$$\text{Sensitivity} = \frac{TP}{TP+FN}$$

$= \text{Fraction of people who have the disease who test positive}$

$$\text{Specificity} = \frac{TN}{TN+FP}$$

$= \text{Fraction of people who do not have the disease who test negative}$

$$\text{Positive Predictive Value} = \frac{TP}{TP+FP}$$

$= \text{Probability patient is actually abnormal when diagnosed as abnormal}$

$$\text{Negative Predictive Value} = \frac{TN}{TN+FN}$$

$= \text{Probability patient is actually normal when diagnosed as normal.}$

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$$\text{True Positive Fraction} = \frac{TP}{TP+FN}$$

$= \text{Sensitivity}$

$= \text{Probability of Detection}$

$$\text{False Positive Fraction} = \frac{FP}{FP+TN}$$

$= 1 - \text{Specificity}$

$= \text{Probability of False Alarm}$

Receiver operating characteristic (ROC) curve plots True Positive Fraction vs. False Positive Fraction

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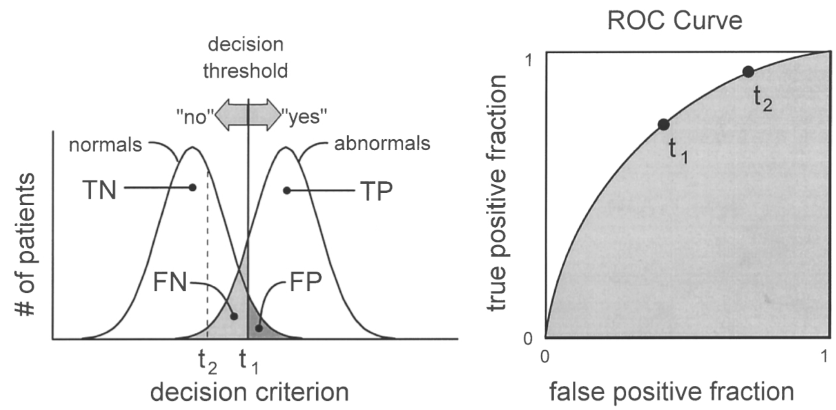
Detection



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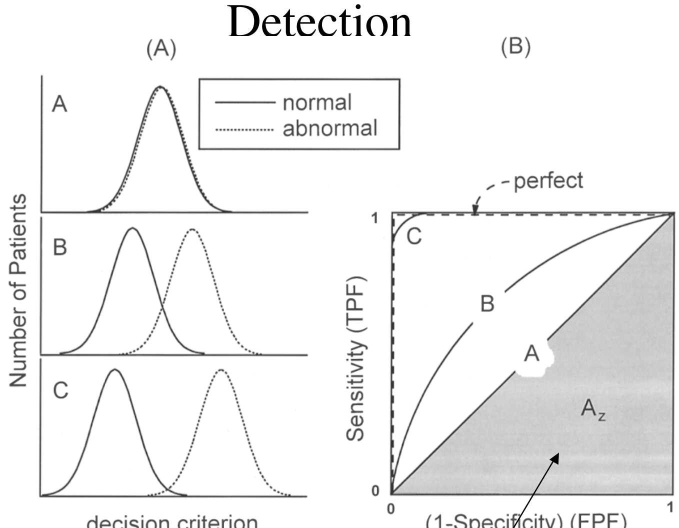
Detection



Area is a measure of detectability

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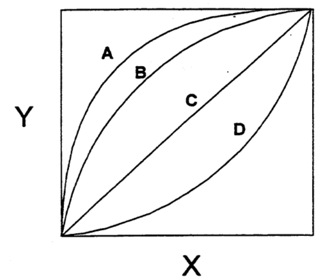
Figure 5:

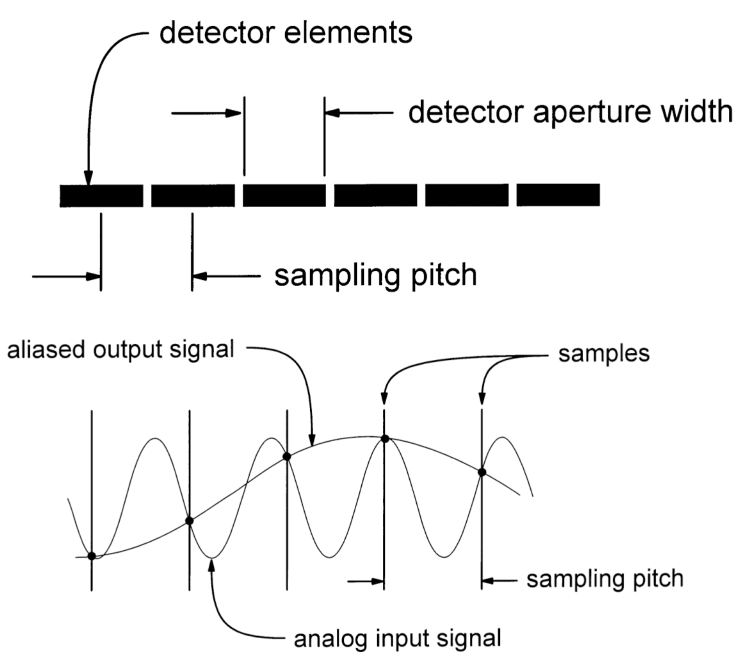


26. In Figure 6, showing an ROC curve, the X axis should be labeled (circle all that are correct)
    A. True Positive Fraction
    B. False Positive Fraction
    C. Sensitivity
    D. Specificity
    E. 1 - Specificity

27. In Figure 6 showing the ROC curves, the Y axis should be labeled (circle all that are correct):
    A. True Positive Fraction
    B. False Positive Fraction
    C. Sensitivity
    D. Specificity
    E. 1 - Specificity

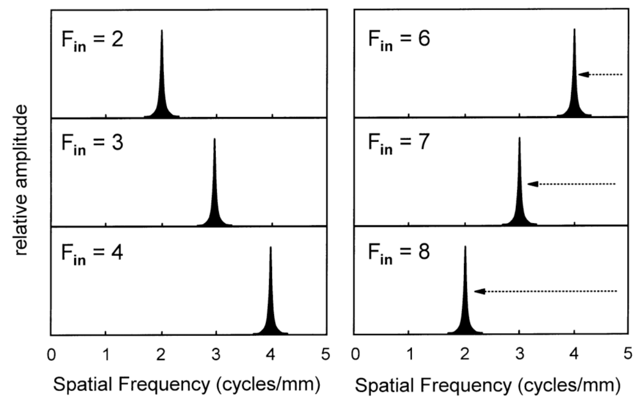
Of the ROC curves in Figure 6,

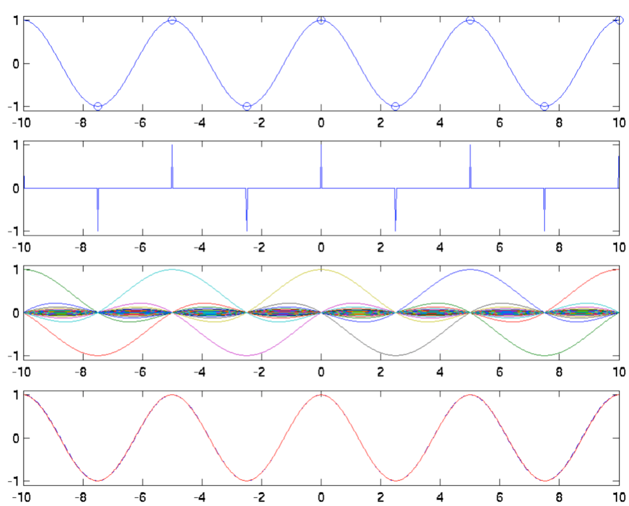
28. Curve number _____ represents pure guessing
29. Curve number _____ represents the best diagnostic approach.
30. Curve number _____ represents an Az value of about 0.3



Nyquist Frequency $= F_N = \dfrac{1}{2\Delta}$ ← Sampling Pitch

If $f > F_N$, then aliasing will occur





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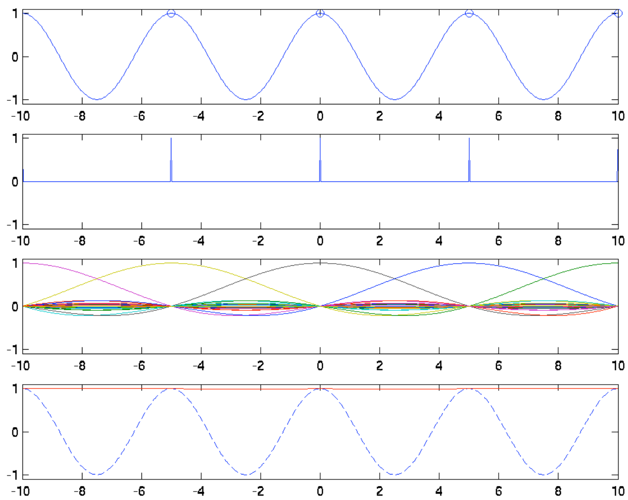
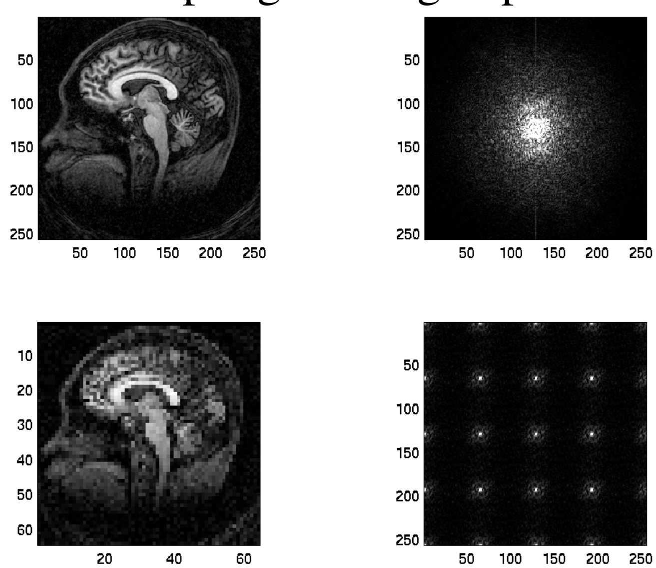
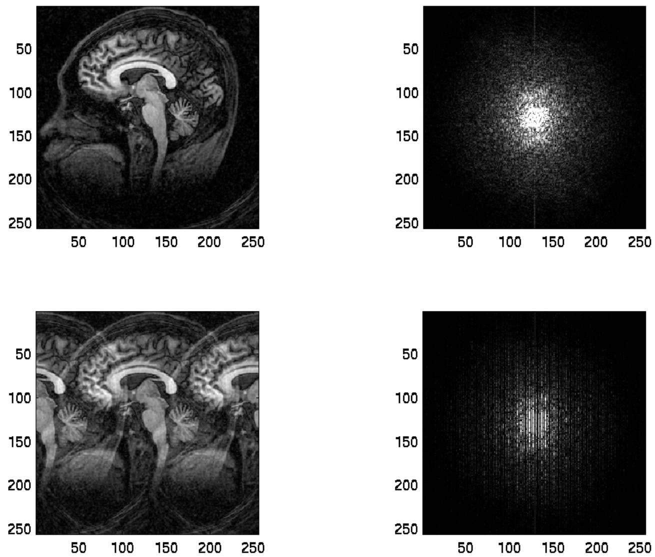
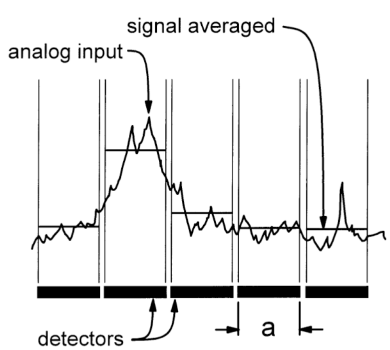


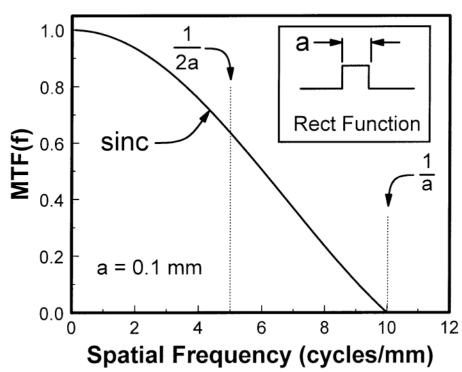
Sampling in Image Space



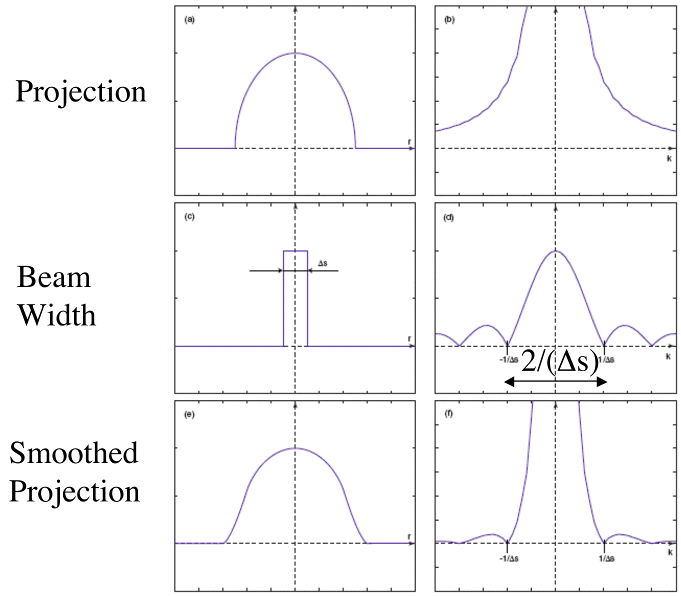
Sampling in k-space







Smoothing of Projections in CT



W= 2/(Δs)
δ=1/W= Δs/2

Suetens 2002